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LINEAR PERSPECTIVE:

OR, A

New METHOD

Of Repreſenting juſtly all manner of

OBJECTS

As they appear to the EYE

IN ALL

SITUATIONS.

A Work neceſſary for PAINTERS, ARCHITECTS, *&c.* to Judge of, and Regulate Deſigns by.

BY

Brook Taylor, LL.D. and R. S. S.

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TO THE READER.

IN this Treatise I have endeavour'd to render the Art of PERSPECTIVE more general, and more easy, than has yet been done. In order to this, I find it necessary to lay aside the common Terms of Art, which have hitherto been used, such as Horizontal Line, Points of Distance, &c. and to use new ones of my own; such as seem to be more significant of the Things they express, and more agreeable

greeable to the general Notion I have formed to my ſelf of this Subject.

Thus much I thought neceſſary to ſay by way of Preface; *becauſe it always needs an Apology to change Terms of Art, or any way to go out of the common Road. But I ſhall add no more, because the ſhortneſs of the Treatiſe it ſelf makes it needleſs to trouble the Reader with a more particular Account of it.*

LINEAR PERSPECTIVE.

SECTION I.

Containing an Explanation of thoſe things that are neceſſary to be underſtood in order to the Practice of PERSPECTIVE.

PERSPECTIVE is the Art of drawing on a Plane the Appearances of any Figures, by the Rules of Geometry.

In order to underſtand the Principles of this Art, we muſt conſider, That a Picture painted in its utmoſt degree of Perfection, ought ſo to affect the Eye of the Beholder, that he ſhould not be able to judge, whether what he ſees be only a few Colours laid artifici-

ally on a Cloth, or the very Objects there represented, seen thro' the Frame of the Picture, as thro' a Window. To produce this Effect, it is plain the Light ought to come from the Picture to the Spectator's Eye, in the very same manner, as it would do from the Objects themselves, if they really were where they seem to be; that is, every Ray of Light ought to come from any Point of the Picture to the Spectator's Eye, with the same Colour, the same strength of Light and Shadow, and in the same Direction, as it would do from the corresponding Point of the real Object, if it were placed where it is imagined to be. So that (*Fig.* 1.) if E F be a Picture, and *a b c d* be the Representation of any Object on it, and A B C D be the real Object placed where it should seem to be to a Spectator's Eye in O; then ought the Figure *a b c d* to seem exactly to cover the Figure A B C D, and the Rays A O, B O, C O, *&c.* that go from any Points A, B, C, *&c.* of the original Object to the Spectator's Eye O, ought to cut the Picture in the corresponding Points *a*, *b*, *c*, *&c.* of the Representation. Wherefore, in order to demonstrate any Proposition in this Treatise, upon this Principle, I always suppose the real original Object to be placed where it should appear to be.

DEFINITIONS.

DEF. I.

THE Center of the Picture is that Point where a Line from the Spectator's Eye cuts it (or its Plane continued beyond the Frame, if need be) at Right Angles.

If the Plane C D (*Fig.* 2.) be the Picture, and O the Spectator's Eye, then a Perpendicular let fall on the Picture from O, will cut it in its Center P.

DEF.

DEF. II.

The Distance of the Picture, or principal Distance, is the Distance between the Center of the Picture and the Spectator's Eye.

In the same Figure P O is the Distance of the Picture.

DEF. III.

The Intersection of an Original Line is that Point where it cuts the Picture.

If I K be an Original Line cutting the Picture in C, then is C the Intersection of the Line I K.

DEF. IV.

The Intersection of an Original Plane, is that Line wherein it cuts the Picture.

A B is an Original Plane cutting the Picture in the Line C Q, which therefore is its Intersection.

DEF. V.

The Vanishing Point of an Original Line, is that Point where a Line passing thro' the Spectator's Eye, parallel to the Original Line, cuts the Picture.

Such is the Point V, the Line O V being parallel to the Original Line I K.

COROL. 1.

Hence it is plain, that Original Lines, which are parallel to each other, have the same Vanishing Point. For one Line passing thro' the Spectator's Eye, parallel to them all, produces the Vanishing Point of 'em all, by this Definition.

COROL. 2.

Those Lines that are parallel to the Picture have no Vanishing Points. Because the Lines which should produce the Vanishing Points, are in this Case also parallel to the Picture, and therefore can never cut it.

COROL. 3.

The Lines that generate the Vanishing Points of two Original Lines, make the same Angle at the Spectator's Eye, as the Original Lines do with each other.

DEF. VI.

The Vanishing Line of an Original Plane, is that Line wherein the Picture is cut by a Plane passing thro' the Spectator's Eye parallel to the Original Plane.

Such is the Line VS, the Plane EF, being parallel to the Original Plane AB.

COROL. 1.

Hence Original Planes, that are parallel, have the same Vanishing Line. For one Plane passing thro' the Spectator's Eye, parallel to them all, produces that Vanishing Line.

COROL.

COROL. 2.

All the Vaniſhing Points of Lines in parallel Planes, are in the Vaniſhing Line of thoſe Planes. For the Lines that produce thoſe Vaniſhing Points, (by *Def.* 5.) are all in the Plane that produces that Vaniſhing Line, (by this *Def.*)

COROL. 3.

The Planes which produce the Vaniſhing Lines of two Original Planes, being parallel to the Original Planes, and paſſing both thro' the Spectator's Eye, (by this *Def.*) have their common Interſection paſſing thro' the Spectator's Eye, parallel to the Interſection of the Original Planes, and are inclined to each other in the ſame Angle as the Original Planes are. And hence,

COROL. 4.

The Vaniſhing Point of the common Interſection of two Planes, is the Interſection of the Vaniſhing Lines of thoſe Planes.

COROL. 5.

The Vaniſhing Line, and Interſection of the ſame Original Plane, are parallel to each other. Becauſe they are generated by parallel Planes. (By this *Def.* and *Def.* 4.)

DEF. VII.

The Center of a Vaniſhing Line is that Point where it is cut by a Perpendicular from the Spectator's Eye.

Such is S, O S being perpendicular to the Vaniſhing Line V S.

COROL.

A Line drawn from the Center of the Picture, to the Center of a Vanishing Line, is perpendicular to that Vanishing Line. As in the Line PS, to the Vanishing Line SV.

DEF. VIII.

The Distance of a Vanishing Line is the Distance between its Center, and the Eye of the Spectator. As OS.

COROL.

The Distance of a Vanishing Line is the Hypothenuse of a Right-Angled Triangle, (such as OPS) whose Base being the Principal Distance (OP) its Perpendicular is the Distance (PS) between the Center of the Picture, and the Center of that Vanishing Line.

DEF. IX.

The Directing Plane is a Plane passing thro' the Spectator's Eye parallel to the Picture.

Such is the Plain GH.

DEF. X.

The Directing Point of an Original Line is that Point where it cuts the Directing Plane.

Such is the Point G, to the Original Line IK.

PROP.

PROP. I. THEOR. I.

The Representation of a Line is Part of a Line passing thro' the Intersection and Vanishing Point of the Original Line.

(*Fig.* 2.) For the visual Rays I O, K O, *&c.* which produce the Representations of the Points I, K, *&c.* of the Original Line I K, by their Intersections with the Picture in *i*, *k*, *&c.* are all in a Plane passing thro' the Original Line I K, and the Spectator's Eye O. But the Point C, which is the Intersection of the Original Line I K, is in that Plane, because it is in the Line I K; and the Line O V is in the same Plane, because it is parallel to the Original Line I K, (by *Def.* 5.) Wherefore the Line V C is the Intersection of the Plane I O K with the Picture, and consequently *i k*, which is the Representation of the Line I K, is Part of the Line V C, which passes thro' the Vanishing Point V, and the Intersection C, of the Original Line I K.

COROL. I.

Hence the Representations of any Number of Lines that are parallel to each other, but not parallel to the Picture, will pass thro' the same Point. For they all pass thro the same Vanishing Point. (By this *Theor.* and by *Cor.* I. *Def.* 5.) See this express'd in *Fig.* 3.

COROL. 2.

But if the Original Lines are parallel to the Picture, as well as to each other, their Representations will be parallel to each other, and to the Originals. For the Line passing thro' the Spectator's Eye, which in other Cases produces the Vanishing Point by its Intersection with the Picture, is in this Case parallel to it, and therefore produces no Vanishing Point. So that the Representations can never meet each other; nor that

that Line passing thro' the Spectator's Eye, and consequently are parallel to each other, to that Line, and to the Originals. See this represented in *Fig.* 4. where A B is the Picture, O the Eye of the Spectator, *&c.*

COROL. 3.

And hence it appears that the Representations of plain Figures, parallel to the Picture, are exactly of the same Shape as their Originals. For (*Fig.* 5.) the Picture being A B, the Original Figure parallel to it being C D E F, and the Representation being *c d e f*, if you resolve the Original Figure into Triangles, by means of Diagonal Lines, such as D F, the Representation will be resolved into Triangles by corresponding Diagonals *d f*. Whence all the Lines in the Representation being parallel to all the Lines in the Original, every Triangle *e d f* will be like the corresponding Triangle E D F, and consequently all the Lines of the Figure *c d e f* will be in the same Proportion to one another, as the corresponding Lines in the Figure C D E F.

PROP. 2. THEOR. 2.

Any Line in the Representation of a Figure parallel to the Picture, is to its Original Line, as the Principal Distance is to the Distance between the Spectator's Eye, and the Plane of the Original Figure.

Let OG (*Fig.* 5.) be perpendicular to the Original Plane, and to the Picture, cutting them in G, and *g*. The Original Plane being parallel to the Picture, therefore all the visual Rays O C, O D, O E, *&c.* are cut in the same Proportion by the Points *c d e*, *&c.* as OG is by the Point *g*; and *d c* being parallel to D C, the Triangle *d* O *c* is like the Triangle D O C; wherefore *dc* is to D C, as O *d* is to O D, that is as O *g* (which is the principal Distance) is to O G.

PROP. 3. THEOR. 3.

The Representation of a Line is parallel to a Line passing thro' its Directing Point and the Spectator's Eye.

For the Directing Point G, (*Fig.* 2.) and consequently the Line O G, is in the Plane I O K, which produces the Representation *i k*. (By *Prop.* 1.) Wherefore the Directing Plane being parallel to the Picture, (by *Def.* 9.) the Representation *i k* is parallel to O G.

COROL.

Hence it appears that Original Lines which pass thro' the same Point of the Directing Plane, have parallel Representations. Which also is true, when they don't pass thro' the same Point of the Directing Plane, but thro' any Points of the same Line (O G) that passes thro' the Spectator's Eye.

PROP. 4. THEOR. 4.

The Distance between the Vanishing Point of a Line and the Representation of any Point in it, is to the Distance between the Vanishing Point and Intersection, as the Distance between the Directing Point and the Intersection, is to the Distance between the Directing Point and the Original Point.

(*Fig.* 2.) Every thing remaining as is explained in the Definitions, and *k* being the Representation of K; the Line V C being Parallel to O G, the Triangles O V *k*, and K G O are similar; wherefore V *k* : O G :: O V : G K. But O G = V C, and O V = G C; wherefore V *k* : V C :: G C : G K.

COROL.

If you imagin a Plane to pass through K parallel to the Picture, this Proportion will be the same as in *Prop*. 2.

SECTION II.

Propositions relating to the General Practice of PERSPECTIVE.

PROP. 5. PROB. 1.

Having given the Center and Distance of the Picture, to find the Representation of a Point, whose Seat on the Picture, and Distance from it are given.

Fig. 6. S is the Center of the Picture, and C the Seat of the Original Point. Thro' S and C draw at pleasure two parallel Lines, and make S O equal to the Distance of the Picture, and C P equal to the Distance of the original Point from it's Seat, and draw O P, which will cut S C in *p*, which is the Point sought.

When the Picture lies between the Spectator's Eye, and the original Point, *p* will fall between S and C; but when the original Point falls between the Spectator's Eye and the Picture, then will *p* fall beyond C, as is expressed by the Letters P1 and *p*1; but when the Spectator's Eye falls between the Picture and the original Point, that Point cannot be seen on the Picture, because it is behind the Spectator, though it's Projection is of use in some Cases.

DEMON-

DEMONSTRATION.

For you may imagin O to be the Spectator's Eye, and P to be the original Point, in their proper Places, then will O P be a visual Ray projecting the Point p on the Picture.

The Point p may be found likewise by Calculation, for $Sp : SC :: OS : OS + CP$, or $Sp1 : SC :: OS : OS - P1 C$.

Prop. 6. Prob. 2.

Having given the Center and Distance of the Picture, and the Position of an Original Line with respect to the Picture, to find its Indefinite Representation and Vanishing Point.

I. *By the Seat, Intersection, and Inclination of the Original Line.*

Fig. 7. Let S be the Center of the Picture, C the Intersection of the original Line, and C D its Seat on the Picture.

Draw C A, so that the Angle A C D may be equal to the Angle the original Line makes with its Seat C D. Draw S V parallel to C D, and perpendicular to it draw S O equal to the Distance of the Picture, and draw O V parallel to C A cutting S V in V, and draw C V. Then will V be the Vanishing Point, and C V the indefinite Representation of the Original Line.

II. *By the Seats of two Points of the Original Line.*

Let D and E (*Fig.* 7.) be the Seats of two Points of the Original Line on the Picture. Draw D E, and S V parallel to it. Perpendicular to D E take D A, and E B equal to the Distances of the two Original Points from

their Seats; and Perpendicular to SV take SO equal to the Distance of the Picture, and draw OV parallel to AB, cutting SV in V. Then will V be the Vanishing Point. Draw SE, and find the Representation of B. (By *Prop.* 5.) Then will V *b* be the indefinite Representation. Or find the Representations *a* and *b* of the two Points A and B (by *Prop.* 5.) and draw *b a* cutting SV in V. Then will V *b* be the indefinite Representation, and V the Vanishing Point.

The Vanishing Point V, may likewise be found by a Line of Tangents; for the Angle VOS is equal to CAD which is the Compliment of the given Angle ACD; wherefore the Distance of the Picture SO being Radius, SV will be the Co-tangent of the Angle ACD.

DEMONSTRATION.

Suppose the Triangles SVO and CDA to be turn'd round the Lines SV, and CD, till O co-incides with the Spectator's Eye, and CA co-incides with the Original Line. Then the Planes SOV, and CAD being parallel to each other, and the Angles OVS, and ACD still equal, OV will be parallel to CA. Wherefore V will be the Vanishing Point, (by *Def.* 5.) and CV will be the indefinite Representation. (By *Prop.* 1.)

PROP. 7. PROB. 3.

Having given the Center and Distance of the Picture, and the Position of an Original Plane, to find its Vanishing Line, its Center, and Distance.

Find the Vanishing Points of two Lines in that Plain, (by *Prop.* 6.) and a Line passing through them will be the Vanishing Line sought. (By *Cor.* 2. *Def.* 6.) Or let S be the Center of the Picture, (*Fig.* 8.) and AB the Intersection of the Original Plane. Draw SO parallel to AB, and equal to the Distance

Distance of the Picture, and draw S V perpendicular to it, and let C be the Angle of the Inclination of the Original Plane to the Picture. Draw O V cutting S V in V, so that the Angle O V S may be equal to C, and draw V D parallel to A B. Then will V D be the Vanishing Line sought, V its Center, and V O its Distance.

The Distance S V may also be found by a Line of Tangents; for S O being Radius, S V is the Co-tangent of C.

DEMONSTRATION.

Turn the Triangle S V O round the Line S V, till its Plane becomes perpendicular to the Picture. Then will the Plane O V D be parallel to the Original Plane, and consequently will produce the Vanishing Line D V. (By *Def.* 6.) And O V being perpendicular to V D, V will be its Center, and V ~~D~~ O its Distance, (by *Def.* 7. and 8.)

PROP. 8. PROB. 4.

Having given the Center and Distance of the Picture, and the Vanishing Line of a Plane, to find the Vanishing Point of Lines perpendicular to that Plane.

Fig. 9. S is the Center of the Picture, and V A the Vanishing Line given. Draw S O parallel to V A, and equal to the Distance of the Picture, and draw S V perpendicular to V A cutting it in V. Then draw V O, and O P perpendicular to it, which will cut V S in the Point sought P.

Or you may find the Distance S P by Calculation, for V S : S O :: S O : O P.

DEMON-

DEMONSTRATION.

If the Plane V O P be turn'd round the Line V P, till O co-incides with the Spectator's Eye; then will O V A be th Plane producing the Vanishing Line V A, and O P being per pendicular to that Plane, will be parallel to the Origina Lines that are perpendicular the Original Planes, and therefor will produce their Vanishing Point P.

N. B. *When the Vanishing Line passes thro' the Center of th Picture, the Points S and V will be all one, and the Distanc S P, or V P will be Infinite, and the Representations of al Lines perpendicular to the Original Plane, will be perpendi cular to the Vanishing Line, and consequently parallel t each other, as they are to the Originals.*

Prop. 9. Prob. 5.

Having given the Center and Distance of the Pi Eture, and the Vanishing Point of a Line; to fin the Vanishing Line of Planes perpendicular to tha Line.

Fig. 9. S is the Center of the Picture, and P the Vanish ing Point given. Draw S P, and perpendicular to it take S O equal to the Distance of the Picture, and draw O P, and
V perpendicular to it draw O ~~K~~ cutting P S in V. Then draw V A perpendicular to V P, and that will be the Vanishing Line sought, V its Center, and V O its Distance.

This is demonstrated as the foregoing *Prop.* by bringing O to the Spectator's Eye. And S V may in the same manner be found by Calculation.

N. B. *When*

N. B. *When the Point P co-incides with the Center of the Picture, the Distance P V, or S V will be Infinite, and there will be no Vanishing Line, the Original Planes being parallel to the Picture.*

Prop. 10. Prob. 6.

Having given the Center and Distance of the Picture, and the Vanishing Line of a Plane, and the Vanishing Point of the Intersection of that Plane, with another Plane perpendicular to it; to find the Vanishing Line of that other Plane.

Fig. 9. S is the Center of the Picture, A V the Vanishing Line given, and in it (by *Cor.* 4. *Def.* 6.) D is the Vanishing Point given. Find P the Vanishing Point of Lines perpendicular to the Plane, whose Vanishing Line is D A. (By *Prop.* .) Then draw D P, which is the Vanishing Line sought.

DEMONSTRATION.

For the Plane whose Vanishing Line is sought being perpendicular to the other Plain, P will be the Vanishing Point of one Line in it. But D is the Vanishing Point of another Line in it by the Supposition. Wherefore D P is its Vanishing Line.

N. B. *When the Vanishing Line A D passes through the Center of the Picture, the Distance V P will be Infinite,* (*by* Prop. 8.) *Wherefore in that Case D P will be parallel to V P, and consequently perpendicular to A D.*

Prop.

PROP. 11. PROB. 7.

Having given the Center and Distance of the Picture, and the Inclination of two Planes, and the Vanishing Line of one of them, together with the Vanishing Point of their Intersection, to find the Vanishing Line of the other Plane.

Fig. 10. S is the Center of the Picture, AB the given Vanishing Line of one of the Planes, and A the Vanishing Point of its Intersection with the other Plane, the Planes being inclined to each other in the Angle C. Find the Vanishing Line TB of a Plane perpendicular to the Line whose Vanishing Point is A, (by *Prop.* 9.) its Center will be V, where it is cut by AS. Take VO equal to the Distance of the Vanishing Line BT, and thro' the Point B, where it cuts the given Vanishing Line AB, draw BO, and make the Angle BOT equal to C, the Line OT cutting TB in T, and draw AT, which will be the Vanishing Line sought.

DEMONSTRATION.

Turn the Triangle BOT round the Line BT, till O coincides with the Spectator's Eye; then will OA be perpendicular both to OB and to OT, it being perpendicular to the Plane BOT by the Construction, and OA will be parallel to the Intersection of the Original Planes. Wherefore AB being the Vanishing Line of one of them, and the Angle BOT being equal to C, TA will be the Vanishing Line of the other of them. (By *Cor.* 3 and 4. *Def.* 6.)

N. B. *When the Intersection of the Planes is parallel to the Picture, the Point A being at an infinite Distance, S A will be parallel to the Vanishing Line B A, and V will co-incide with S, and B V will be perpendicular to B A; V O will be the Principal Distance, and T A will be parallel to B A.*

PROP.

Prop. 12. Prob. 8.

Having given the Vanishing Line of a Plane, its Center and Distance, and the Vanishing Point of a Line in that Plane, to find the Vanishing Point of Lines that make a given Angle with that Line.

Q V (*Fig.* 11.) is the Vanishing Line given, S its Center, and V the Vanishing Point given. Perpendicular to Q V draw S O equal to the Distance given. Then draw O V and make the Angle V O Q equal to the given Angle, and Q will be the Vanishing Point sought. So that if you draw any two Lines V P, and Q P, the Angle V P Q will represent an Angle equal to V O Q.

DEMONSTRATION.

Turn V O Q round the Vanishing Line V Q, till O co-incides with the Spectator's Eye. Then will O V and O Q be parallel to the Original Lines whose Vanishing Points are V and Q, and consequently the Angle they make with each other is equal to V O Q.

By this Proposition you may find the Representations of any plain Figures, having one side. For the Original Figure may be resolved into Triangles, whose Angles being all given, the Vanishing Points of all their sides will be given by this Proposition. Whence the Representations of all the sides will be found, beginning first with those that lie next the side given.

Here likewise you may observe, That the Center of the Picture is not concerned in this *Problem.* Wherefore to see the Representations of Figures in a Plane, whose Vanishing Line is V Q, the Spectator's Eye may be placed any where in the Circumference of a Circle described by O, while the Plane V O Q turns round the Axis V Q.

Prop. 13. Prob. 9.

Having given the Repreſentation of a Line, and it Vaniſhing Point; to find the Repreſentation of Point that divides the Original Line in a give Proportion.

Fig. 12. AB is the Repreſentation given, and V the Va niſhing Point. Draw at pleaſure A *b*, and divide it in th given Proportion in *c*. Draw V O parallel to A *b*, and dra *b* B cutting it in O. Then draw O *c*, which will cut A B i the Point ſought C.

This may likewiſe be done by Calculation; for AC : $AB :: Ac \times AV : Ac \times AV + bc \times BV$, or AC $: AB :: Ac1 \times AV : Ac1 \times AV - bc1 \times BV$, or $AC2 : AB :: Ac2 \times AV : bc2 \times BV - Ac$ $\times AV$.

DEMONSTRATION.

Suppoſe the Original Line to be in a Plane, whoſe Vaniſh ing Line is V O. Then will O A, O B *b*, and O C *c* re preſent parallel Lines. Wherefore the Originals of A B and A *b* are divided in the ſame Proportion by the Originals o the Points C and *c*. But A *b* being parallel to V O, it is di vided by *c* in the ſame Proportion as its Original (by *Cor.* *Prop.* 1.) that is, in the Proportion given. Wherefore the O riginal of A B is alſo divided in the Proportion given.

Prop

Prop. 14. Prob. 10.

Having given the Vanishing Point of a Line V (Fig. 13.) and three Points in its Representation, A, B, C; to find a fourth Point D, so that the Part represented by C D may be to the Part represented by A B, in a given Proportion.

Draw at pleasure V O, and A d parallel to each other, and thro' any Point O of the Line V O draw V B, V C cutting A d in b, and c. Make $c\,d$ to A b in the given Proportion, and draw O d cutting A V in D, which will be the Point sought.

This is demonstrated as the foregoing *Proposition.*

The Point D may also be found by a Scale and Compasses, by the help of this Proportion $CD : CV :: cd \times AB \times CV : cd \times AB \times CV + Ab \times AV \times BV$. Which may easily be accommodated to any Position of the Points A, B, C, D, only observing well the Signs + and —.

Prop. 15. Prob. 11.

Having given the Vanishing Line of a Plane, its Center and Distance, and the Representations of two Lines in that Plane, from a given Point in one of them to cut off a Line, that shall represent a Line in a given Proportion to the Line represented by the other.

Fig. 14. S D is the given Vanishing Line, and V its Center, and A B and C I are the Representations given, whose Vanishing Points are D and E. It is required to find the Point I, so that the Line represented by C I, may be to the Line represented by A B, as c is to d. Draw V O perpendicular to F D,

FD, and equal to the Distance given, and draw OD and OE, and make OQ : OR : : c : d, and draw OS parallel to RQ. Draw AC cutting SD in F, and draw FB and DC cutting each other in p. Then draw pS, which will cut CE in the Point sought I.

DEMONSTRATION.

Because of the Vanishing Points D and F, the Figure AB pC represents a Parallelogram. Wherefore Cp represents a Line equal to the Line represented by AB.

By the manner of placing the Point O, the Plane SOD may be considered as the Plane producing the Vanishing Line SD, O being the Spectator's Eye. Wherefore OS being parallel to RQ, D, E and S are the Vanishing Points of the Sides of a Triangle parallel to ROQ. Wherefore pCI represents that Triangle, and consequently the Line represented by CI is to the Line represented by pC, or by AB, as OQ is to OR; that is, as c is to d.

If the Point F falls out of reach, you may bring it into the bounds of the Picture thus. Instead of AB suppose ab to be one of the Lines given, whose Vanishing Point is D. Draw DA, and taking any Point r in the Vanishing Point FD, as is most convenient, draw ra and rb cutting DA in A and B, and use the Line AB, as above, instead of ab. For because of the Vanishing Points r and D, AB and ab represent equal Lines.

PROP. 16. PROB. 12.

Having given the Vanishing Line of a Plane, and the Representations of two Lines in it; to find a Point in one of them, so that it may represent a Line divided in the same Proportion, as the Line represented by the other.

Fig. 15. P *k* is the Vaniſhing Line given, and AC, DE are the two Repreſentations, and it is required to find the Point F, ſo that the Parts repreſented by DF and FE may be in the ſame Proportion as the Parts repreſented by AB and BC. Draw any Line *s v* parallel to the Vaniſhing Line, and by means of any Vaniſhing Point *w* transfer the Points A, B, C, to *s*, *t*, *v*, and *s v* will be divided in the ſame Proportion as the Original of AC (by *Cor*. 3. *Prop*. 1.) and becauſe of *w s*, *w t*, *w v*, repreſenting Parallels. Whence you may find the Point F by *Prop*. 13.

When the Vaniſhing Point *k* of the Line AB falls within reach, you may find the Proportion of *s t*, to *t v*, without drawing any Lines on the Picture. For *s t* : *t v* : : AB × C *k* : BC × A *k*.

By theſe Propoſitions you may find the Perſpective Repreſentation of any Figures propoſed. For you may conſider the whole Deſign as one Figure, the Poſition of whoſe Parts are all given with reſpect to each other. Then having found the Vaniſhing Line of ſome Principal Plane, and the Repreſentation of a Remarkable Line in the Deſign, by *Prop*. 5, 6, 7, you may find the Vaniſhing Lines of all other Plains, by their Poſitions with reſpect to the Plane firſt aſſumed, by *Prop*. 9, 10, 11. And then by the Vaniſhing Points already given, and by the given Proportions of the Parts, you may find the Repreſentations of the Figures propoſed by *Prop*. 12, 13, 14, 15, 16. Or having given the Plan of any Figure, upon any Plane that is conveniently choſen, and the Elevations of the Parts of the Figure above that Plane, having found the Repreſentation of the Plan, by *Prop*. 12, or by that and *Prop*. 13, 14, 15, 16, you may then find the Repreſentations of the Elevations, by the help of *Prop*. 8. And in order to do all this, there is no neceſſity of having an Original Deſign drawn out in its juſt Proportions; but it is ſufficient to have the Proportions of the Parts expreſs'd in Numbers; the Deſign being any how ſcetch'd out to help the Memory. This I ſhall illuſtrate by a few *Examples*.

EXAM-

EXAMPLE I.

To find the Representations of any Figures on a Plane having given the Intersection, and Vanishing Line, Center and Distance, the Original Plane being drawn out in its just Proportions.

At the end of *Prop.* 12. I have observed that the Shapes of the Representations of Figures on a Plane don't at all depend upon the Angle the Picture makes with that Plane. Wherefore let F E (*Fig.* 16.) be the Intersection given, *f e* the Vanishing Line, V its Center, and V O its Distance perpendicular to it. Then O being consider'd as the Spectator's Eye in the Plane X, which generates the Vanishing Line, and which is now so turn'd as to co-incide with the Plane of the Picture Y, let Z be the Original Plane turned in the same manner till it co-incides with the Picture, the Figures A B C D and M being now seen on the back-side. Then to find the Representation of any Point B, draw B E cutting E F in E, and draw O *e* parallel to it, cutting the Vanishing Line in *e*; then E being the Intersection, and *e* the Vanishing Point of the Line B E, the Representation of the Point B is found by drawing E *e*, and the visual Ray O B cutting it in B. The Point C is found without drawing the visual Ray, by the Intersection of the indefinite Representations E *e*, and F *f* of the Lines C E and C F. And in the same manner the Representation *m* of the Figure M, is found by the Intersections and Vanishing Points, without drawing any visual Rays.

Or you may find the Representations of the Figures proposed by means of the Directing Plane. Thus (*Fig.* 17.) D F C being the Original Plane reversed, as before, D F G H the Plane of the Picture, D F the Intersection, G H the Vanishing Line, V its Center, and V O its Distance set off perpendicular to it, as before; having drawn *d f* parallel to G H, and as far below O, as D F is below G H, *d* O F will be the Directing Plane brought into the Plane of the Picture, *d f* being its Intersection with the Original Plane. Whence having continued

continued the Sides of the Triangle A B C, till they cut D F and *d f* in D, E, F, and *d*, *e*, *f*; D, E and F being their Intersections, and *d*, *e*, *f*, their Directing Points, drawing D *b* parallel to O *d*, E *c* parallel to O *e*, and F *c* parallel to O *f*, they by their Intersections will give the Representation sought *a b c*, (by *Prop*. 3.)

EXAMPLE II.

To find the Representation of any given plane Figure, having given the Representation of one Side of it, and the Vanishing Line, Center and Distance of the Plane it is in. (*Fig*. 18.) M F is the given Vanishing Line, V its Center, and V O its Distance set off perpendicular to it, and *a b* is the Line given to represent the Side AB of the Figure ABCDE. Having resolved the Original Figure into Triangles, by means of the Diagonals A C, A D, the Vanishing Points of the Sides A C, B C, are found by continuing *a b* till it cuts the Vanishing Line in its Vanishing Point F; and then making the Angle F O H equal to B A C, you have the Vanishing Point H of the Side A C; and by making the Angle F O G equal to *p* B C, you have the Vanishing Point G of the Side B C: Whence you have the Representation *a b c* of the Triangle A B C. And in the same manner by the Side *a c* you will get the Triangle *a c d*, and then by *a d* you get the Triangle *a d e*, and by that means have the Representation *a b c d e* of the whole Figure. This is founded upon *Prop*. 12.

Or you may proceed thus. Continue the Sides D C, D E (*Fig*. 19.) till they cut A B in P and Q. Then having found the Representations *p* and *q* of the Points P and Q (by *Prop*. 13.) you will find the Representation of the Triangle P D Q, as before (by *Prop*. 12.) and then you will have the Points *c* and *e*, by *Prop*. 13.

Or having continued *a b* (*Fig*. 20.) till it cuts the Vanishing Line in F, and drawn D E parallel to the Vanishing Line H F, cutting *a b* in D, draw D B parallel to O F, and draw the visual Rays O *a*, O *b*, cutting D B in A and B, and on the

the Side A B make a Figure A C B like the Original Figure and proceed as in *Exam.* 1. the Line D E being consider'd as the Intersection, and D C E as the Original Plane invert'd.

Curve-lined Figures are described by finding several Points and then joyning them neatly by Hand. And this may conveniently be done by putting the Geometrical Descriptions of Curves into Perspective, as in the two following Examples of describing a Circle.

EXAMPLE III.

Having given the Vanishing Line of a Plane, its Center and Distance; to find the Representation of a Circle, from the given Representation of one Radius.

Fig. 21. C is the Representation of the Center, and C A of the Radius given, D E is the Vanishing Line, and O the Spectator's Eye placed as in the foregoing Examples. Draw at Pleasure C B, and make C B to represent a Line equal to that represented by C A (by *Prop.* 15.) that is, bisect the Angle E O D by the Line O F, and draw F A, cutting E C in B. And in the same manner you may find as many Points B in the Circumference of the Circle, as you please.

EXAMPLE. IV.

Having given the Vanishing Line of a Plane, its Center and Distance, thro' three Points given to draw the Representation of a Circle.

Fig. 22. A, B, and C are the three Points given, D *f* the Vanishing Line, and O the Spectator's Eye placed as above. Draw C A and C B cutting the Vanishing Line in D and E, and draw D O and E O. Draw any Line *d* O, and make the Angle *d* O *e* equal to D O E, or having made an Instrument containing the Angle D O E, turn it round the Center O, till it comes into the Position *d* O *e*, the Leg O D cutting the

e Vanishing Line in *d*, and OE, (or O *e*) cutting it in *f*. Thro' *d* and *f* draw *d* A and *f* B cutting each other in *p*, and ou will have one Point in the Representation sought. This Construction is taken from the equality of Angles in the Circle sisting upon the same Base, whence the Vanishing Points are und by *Prop.* 13.

EXAMPLE V.

Having given the same things as in either of the two foreoing Examples, from a given Point of the Picture to draw a ine that shall touch the Representation of the Circle.

Fig. 23. DE is the Vanishing Line, O the Eye of the Spectator, C the Representation of the Center, and CA the Reresentation of the Radius, and P the Point given, from hence a Tangent is to be drawn. Draw PC cutting the anishing Line in E, and (by *Exam.* 3.) find the Radius B. Any where apart draw *b p*, and make *b c* : *c p* : : BC PE : CP × BE; that is, make the line *b c p* like the Oginal of BCP. With the Center *c* and Radius *c b* make a ircle, and draw the Tangent *p* T, and then find the Reprentation P *t* of it by *Exam.* 2. And you may proceed in the me manner by finding the Original Figure, when three Points e given.

If instead of from the Point P, it had been required to draw Tangent parallel to CE, the Point P being supposed infiitely distant, PE would be equal to PC, and consequently would be *b c* : *c p* : : BC : BE. But in this Case, as also lways when P falls beyond E, you must take *c p* backwards, falling on the contrary side of the Point *c*, to what it does in e present Scheme. But I shall leave the Reader to consider ll the variety of Cases that may happen from the different Poition of the Points.

The Use of this Example is to find the apparent part of the ase of a Cone or Cylinder.

EXAMPLE VI.

Fig. 24. In this Figure having given the Center of the Picture S, its Distance equal to the Line L, and the Line A B to represent one side of the first Step, and K I the Vanishing Line of the Plane of the Horizon; the Representation of the Base A B C D of the first Step, is found by the Vanishing Points I, K, L, which are found by *Prop.* 12. Then by *Prop.* 8. having found the Vanishing Point M of the Perpendiculars A E, F C, *&c.* the Vanishing Line of the Face A C E F being M K, the Vanishing Point N of the Diagonal A F is found by *Prop.* 12. Whence all the rest of the Lines are found, as appears sufficiently in the Figure.

EXAMPLE VII.

Fig. 25. In this Figure the Representation H I K Q of a regular Tetraedrum is found as follows; Having given the Side H I, the Center of the Picture S, its Distance equal to the Line L, and G F the Vanishing Line of the Face H I K. Having continued the given Side H I till it cuts the Vanishing Line G F, in its own Vanishing Point F, the Vanishing Points G and M of the Sides I K and H K are found by *Prop.* 12. Then having made an Equilateral Triangle A B C, and drawn A D perpendicular to B C, and made the Isosceles Triangle A E C, whose Sides A E, and C E are equal to A D; the Angle A E C being equal to the Inclination of two Faces of the Figure proposed, the Vanishing Line F N of the Face H I Q is found by *Prop.* 11. Whence having the Vanishing Points P and N of the Sides H Q and I Q, you have the whole Representation sought.

The Reader may exercise himself in drawing the Representations of the five regular Solids, in order to which their Plans and Profils may be found in the following manner.

Fig. 26. Having made an Equilateral Triangle A B C, and found its Center G, you have the Plan A B C G of a regular Tetraedron,

Tetraedron. Then making the Triangle B C E, whose Sides B E, and C E are equal to the Perpendicular B D, and drawing C F perpendicular to B E, you have C F equal to the height of the Vertex above the Center G.

It would be superfluous to shew how to make the Plan and elevation of a Cube.

Fig. 27. To find the Plan and elevation of a regular Octaedron, make the regular Hexagon A E B F C D, and having drawn the Lines, as appears in the Figure, you have the Plan of the Figure, on the Plane of one Face A B C. Then having drawn B D cutting A C in G, and D H perpendicular to it, with the Center G and Radius G B make a Circle cutting D H in H, and D H will be the Distance between two opposite Faces.

Fig. 28. To find the Plan of a regular Dodecaedron, make a regular Decagon A B C D E F G H I K, whose Center is S. Then draw a Diagonal A D, leaving out two Angles B and C, and draw S B cutting it in M, and make two regular Pentagons M N O P L, *m n o p l* in sub-contrary Positions, having the same Center S, and draw the Lines N D, O F, P H, L K, *o* A, *p* C, &c. and you will have the Plan of a Dodecaedron on one of its Faces M N O P L.

Draw *a v*, and in it make *a b* and *b v* equal to the Perpendicular L R let fall from one Angle of the Pentagon to the opposite Side, and make the Isosceles-Triangle *c b v*, with the Base *c v* equal to B D, or, which is all one, to a Diagonal L N of the Pentagon. In *v c* take *c e* equal to L M, and make *a d*, *d f*, *e f* parallel and equal to their opposite Lines *c e*, *c b*, *b a*, and let *s* be the Center of the Figure *a b c e f d*. Then bisect *c v* in *g*, and draw *s g* cutting *b c* in *h*, and make *n m* perpendicular to *a d*, *s m* and *s n* being each equal to half L M, and make *b i*, *f k*, *f l* each equal to *b h*, and having drawn the Lines as appears in the Figure, you will have the Profil of the same Figure; *d*, *n* and *h* being the height of the Angles perpendicular over K, B and H, D and F, and *k*, *m*, and *o*

being the Elevations of the Angles perpendicular over A and B, C and G, and E, and *f l e* being the Elevations of the upper Face, which is perpendicular over *n o p l m*.

Fig. 29. To find the Plan and Profil of a regular Icosaedron, make a Right-Angle A B C, B C being the double of A B, and with the Center A and Radius A C make a Circle cutting A B in D. Then make a regular Hexagon E F G H I K, whose Center is S, and having drawn the Lines S E, S F, S G, *&c.* make the Distances S L, S M, *&c.* to S F as B D is to B C, that is, as the Side of a Pentagon is to the Diagonal, and draw all the Lines as appears in the Figure, and you will have the Plan of a Icosaedron on one of its Faces L M N.

To find the Profil of the same Figure make the Isosceles Triangle O P Q, whose Base O Q is equal to the Difference between L M and E G, and the Sides O P, Q P are equal to a Perpendicular let fall from L to the opposite Side M N of the Triangle L M N. In Q P make P R = P O, and in Q O make O T = L M, and make R X, X V, V T parallel and equal to their opposite Lines O T, O P, P R, and draw the Lines as appears in the Figure, and you will have the Profil sought, Z being the Elevation of the Points over F and K, Y that over G and I, R that over H, T that over E, V that over *m* and *n*, and X that over *l*; and observe that the Lines Z R, and T Y are parallel to O P, or V X.

from Q and R: T — X:

EXAMPLE VIII.

Fig. 30. Having given the Center of the Picture S, its Distance S D, and the Representation C A of one Radius of a Sphere, whose Vanishing Point is V, the Representation of the Center being C, the Representation of the Sphere is found as follows. Draw S R perpendicular to A V, and in it take R O equal to the Hypothenuse of a Right-angled Triangle, whose Base is S R, and perpendicular is S D, that is, make R O equal to the Distance of the Spectator's Eye from the Point R, Draw A O, V O, C O, and draw any Line parallel to V O, cutting

cutting A O, and C O in *a* and *c*, and with the Center *c*, and Radius *c a* describe a Circle, to which draw a Tangent O *t* cutting C A in T. Then find the Vanishing Line P Q of a Plane perpendicular to the Line, whose Vanishing Point is C (by *Prop.* 9.) and find the Representation of a Circle in that Plane, the Representation of one Radius being C T, and the Center C, (by *Exam.* 3.) and that will be the Contour of the Sphere.

The Demonstration of this Construction is easy to one that considers, That the visual Rays which touch a Sphere make a Cone, whose Base is a lesser Circle of the Sphere, whose Representation is the Contour of the Representation of the Sphere.

In the Practice of *Perspective* it is often Necessary to draw Lines towards a Vanishing Point, or some other Point that is out of reach, in which case the following Propositions are useful.

Prop. 17. Prob. 13.

From a given Point to draw a Line that shall tend to an inexcessible Point of a given Line, having the Distance of the inaccessible Point from a given Point of that Line.

Fig. 31. Q is the inaccessible Point in the Line P Q, the Distance B Q being given, and A is the Point from whence a Line is to be drawn tending towards Q.

Draw at pleasure A P, cutting P Q in P, and taking the Measure of P B by a Scale of equal Parts, you will have the length of P Q. Take any Point *q* in the Line B Q, and make P *a* : P *q* : : P A : P Q. Draw *a q*, and A Q will be parallel to it.

PROP.

Prop. 18. Prob. 14.

Having given two Lines that tend to an inaccessible Point, thro' a given Point to draw a Line that shall tend to the same Point.

Fig. 32. A B and C D are the given Lines tending to the inaccessible Point, and P is the Point thro' which it is required to draw a Line P *p* tending to the same Point.

Draw at pleasure P A (*n* : 1) and P B parallel to it, cutting A B and C D in C, A, B, and D, and make B *p* : B D :: A P : A C, and draw P *p*, which will be the Line sought.

Otherwise, without the Compasses.

n : 2, and *n* : 3. Thro' P draw at pleasure two Lines cutting A B and C D in A and C, B and D. Then draw A D and B C meeting in Q. Thro' Q draw at pleasure two Lines cutting A B and C D in *b* and *c*, *a* and *d*, and draw *ac* and *b d* meeting in *p*, and draw P *p*, which will be the Line sought.

Section III.

Of finding the Shadows of given Figures.

In this Place I consider Shadows only as the Projections of given Figures on given Surfaces, by the means of given Luminous Points. For I consider the Luminous Body as a Point, to avoid the Difficulties that would attend the Description of Shadows, if the Magnitude of the Luminous Body were taken into Consideration, it being sufficient for Practice to regard only the Center of the Luminous Body; and having found the Contour of the Shadow in that Case, the Penumbra may be drawn

awn by a good Judgment founded on much Observation; being difficult to bring every thing to exact Mathematical onstructions, at least so as to be most convenient for Practice. it the Artist that has made himself well acquainted with the rinciples of this Art, will easily find ways to satisfy himself in ose cases that require more Exactness.

Prop. 19. Prob. 15.

Having given the Center and Distance of the Picture, and the Vanishing Line of a Plane, and the Representation of a Luminous Point, and of its Seat on that Plane; to find the Representation of the Shadow of a Point, whose Representation and Seat on the same Plane are also given; and to find the Vanishing Point of the Ray of Light.

Fig. 33. E T is the given Vanishing Line, S the Representation of the Luminous Point, R the Representation of its eat, A the Representation of the Point whose Shadow is to be ound, B the Representation of its Seat, and P the Vanishing oint of Lines perpendicular to the given Plane, (found by *rop.* 8) Draw R B cutting the Vanishing Line in V, and raw V P; then draw S A cutting R V in C, and P V in D. Then will C be the Shadow sought, and D the Vanishing Point of the Ray of Light. (B)

DEMONSTRATION.

For the Originals of R, B, A, and S are in a plain Perpendicular to the Plane, on which the Shadow is to be cast; wherefore C is the Shadow, it being the Intersection of the Ray S A with the Plane R B. Besides, V is the Vanishing Point of R B. Wherefore P V is the Vanishing Line of R S A B, and consequently D is the Vanishing Point of S A.

N. B. *When*

N. B. *When the Original Luminous Point is behind the Spectator, so that it cannot have any real Representation on the Picture, its imaginary Representation (which is as it were the Shadow of the Spectator's Eye on the Picture,) must be on the contrary Side of the Plane to the Point, whose Shadow is sought, that is, if B be between P and A, S on the contrary must be between P and R (as in* n : 2.) *And when the Luminous Point is the Sun, or any other Light supposed to be at an infinite Distance, the Points R and V will co-incide, and D will be the same as S, as I have shewn in* n : 3, *and* n : 4. *In* n : 3, *the Sun appears in the Picture: But in* n : 4, *it is behind the Spectator, and its imaginary Representation is S.*

Prop. 20. Prob. 16.

Having given the Center and Distance of the Picture, and the Representation of a Line, and of the Seat of one Point of it on a Plane whose Vanishing Line is given, and also the Vanishing Point of that Line; to find the Representation of the whole Seat of that Line on that Plane.

Fig. 34. V E is the given Vanishing Line of the Original Plane, P is the Vanishing Point of Lines perpendicular to it, (found by *Prop.* 8.) A C is the given Representation of a Line, D its Vanishing Point, and B the Representation of the Seat of one Point of it, whose Representation is A. Draw P D cutting V E in E, and V B cutting A D in C. Then will V B be the Representation of the whole Seat of the Line A D.

DEMONSTRATION.

For the Original of A B C is a Triangle whose Vanishing Line is P D. Wherefore V must necessarily be the Vanishing Point of B C, &c.

Whe-

When you have got the Description of any Figure by the foregoing *Propositions*, you generally have the Vanishing Points of all the Lines of it. So that having the Seat of one Point of the Figure on the Plane you intend to cast the Shadow on, you may find the Seats of all the other Points of the Figure by this *Proposition*, and their Shadows by the foregoing.

PROP. 21. PROB. 17.

Having given the Center and Distance of the Picture, and the Vanishing Lines of two Planes, and the Representation of their common Intersection, and the Representation of a Line, and of its Seat on one Plane; to find the Representation of the Seat of that Line on the other Plane.

Fig. 35. A B and C B are the Vanishing Lines of the two Planes, B D the Representation of their common Intersection, E the given Representation of a Line, and A E the Representation of its Seat on the Plane, whose Vanishing Line is A B. By *Prop.* 8. find the Vanishing Points G and H of Lines perpendicular to the Planes, whose Vanishing Lines are A B and C B. Draw A G cutting F E in F, and B C in C, and draw H F cutting B C in I, and draw C D cutting E F in K. Then draw I K, which will be the Line sought.

DEMONSTRATION.

F, A, and C are the Vanishing Points of the Lines E F, E A, and K C, whose Originals are all in the same Plane, because of the Vanishing Line G C, and because they meet in E and D. But because of the Point D being in the Representation of the Intersection of the two Planes, and because of the Vanishing Line B C, the Original of D K, and consequently of the Point K, is in the Plane, whose Vanishing Line is B C. But because of the Vanishing Line H F, I is the

Vanishing Point of the Seat of E F on the Plane, whose Va-
nishing Line is B C; wherefore K being the Representation
of the Intersection of that Line with that Plane, K I will
be the Representation of its Seat.

By this *Proposition* you may carry a Shadow from one Plane
to another. For E F being the Ray of Light, having its Po-
sition with respect to the Plane, whose Vanishing Line is A B,
you find the Shadow K on the Plane, whose Vanishing Line
is B C. So that by these three last *Propositions* you may find
the Shadows of any Figures.

In some Cases perhaps it may be most convenient to find
the Shadows by putting the foregoing Rules of *Perspective*
into *Perspective*. For Example (*Fig.* 36.) G, E, and F
being the Vanishing Points of the Sides of the Triangle ABC
whose Shadow is to be cast on a Plane, whose Vanishing
Line is T V, and Intersection with the Plane of the Tri-
angle A B C is D I; to find the Shadow cast by the Lu-
minous Point represented by S. I seek the Representation
D *g* of the Intersection of a Plane passing through the Lu-
minous Point parallel to the Plane of the Original Tri-
angle, with the other Plane: Then drawing S G, S E,
and S F cutting D *g* in *g*, *e* and *f*, and continuing the Sides
of the Triangle, till they cut D I in H, I, and K; I con-
sider S as if it were a Spectator's Eye, and use *g*, *e* and *f*
as Vanishing Points, and K, H, and I as Intersections of
the Lines B C, B A, A C, and so find the Shadow *abc*

SECT.

SECTION IV.

f finding the Representations of the Reflections of Figures on polish'd Planes.

It is well known that the Reflections of Figures on a po-
h'd Plane, as in a Looking-Glass, or on the Surface of stand-
g Water, appear to be just as much on one side of the
ane, as the real Objects are on the other side. So that to
d the reflected Appearance of any Point, you must draw
Perpendicular to the reflecting Plane from the real Point,
d in it take a Point at the same Distance on the contrary
de of the Plane. For Instance, To find the apparent Place
the Reflection of the Point P (*Fig.* 37.) on the Plane AB draw
e Perpendicular P B, and on the contrary side of the Plane
ake B p = B P, and p will be the apparent Place of the
oint P, seen by Reflection. And from hence it follows that
e Appearance of any Figure seen by Reflection, is exactly
f the same Shape, and of the same Bigness as the real Figure,
it in an inverted Position.

Upon these Principles depend the following *Propositions*, by
hich you may easily find the *Perspective* Representations of
e Appearances of any Figures seen by Reflexion from given
lanes.

PROP. 22. PROB. 18.

Having given the Center and Distance of the Picture and the Vanishing Line of a reflecting Plane, and the Representation of the Seat of a Point on the Plane; to find the Representation of the Reflection of that Point.

Fig. 38. AB is the Vanishing Line given, P the Representation of the real Point, Q the Representation of its Seat on the reflecting Plane, and V the Vanishing Point of Lines perpendicular to that Plane. Make Q*p* to represent a Line equal to that represented by QP (by *Prop.* 13.) and *p* will be the Point sought.

This is evident by the Introduction to this *Section*.

PROP. 23. PROB. 19.

Having given the Center and Distance of the Picture and the Vanishing Line of a reflecting Plane, and the Vanishing Point of a Line, together with the Representation of its Seat on the reflecting Plane; to find the Representation of the Reflection of that Line, and its Vanishing Point.

Fig. 39. AB is the Vanishing Line of the Reflecting Plane, V the Vanishing Point of Lines perpendicular to it, PI the Representation of the real Line, C its Vanishing Point, and BI the Representation of its Seat on the Reflecting Plane. With the Vanishing Point V make B*c*, and BC to represent equal Lines, (by *Prop.* 13.) then will *c* be the Vanishing Point of the Reflection; and drawing *c*I, that will be its Representation.

DEMON-

DEMONSTRATION.

For if you seek the Reflection of the Line by the Reflections of two Points of it, by the foregoing *Proposition*, then drawing any Line V P, the Parts P Q and Q p will represent equal Lines; otherwise p can't represent the Reflection of P. Wherefore when Q co-incides with B, B C and B c will also represent equal Lines. But in that Case C, B, and c, are the Vanishing Points of the real Line P I, its Seat B I, and its Reflection p I. Wherefore the Vanishing Point c, and the Reflection c I are rightly determined by the foregoing Construction.

PROP. 24. PROB. 20.

Having given the Center and Distance of the Picture, and the Vanishing Line of a reflecting Plane; to find the Vanishing Line of the Reflection of a Plane, whose Vanishing Line is also given.

Fig. 39. A B is the Vanishing Line of the Reflecting Plane, V the Vanishing Point of Lines perpendicular to it, and A C the Vanishing Line of the real Plane. Draw V C cutting A C and A B in C and B, and find the Point c, as in the foregoing *Proposition*, and draw A c, which will be the Vanishing Line sought.

DEMONSTRATION.

For by the foregoing *Proposition* c is the Vanishing Point of the Reflection of a Line, whose Vanishing Point is C. But the Vanishing Line A C is the Place of all the Points C; and therefore the Vanishing Line A c is the Place of all the Points c, and consequently is rightly determined by finding one of them, and drawing A c.

By

By theſe three *Propoſitions* having got the Vaniſhing Line of the Reflection of one Face of a given Figure, and the Repreſentation of the Reflection of one ſide of that Face, you may find the whole Reflection as you do the real Figure, by the foregoing *Propoſitions*, only obſerving to make the Reflections in a contrary Poſition to the real Objects.

SECTION V.

Of the Inverſe Practice of PERSPECTIVE, and of the manner of Examining Pictures already drawn.

PROP. 25. PROB. 21.

Having given the Repreſentation of a Line divided into two Parts in a given Proportion; to find its Vaniſhing Point.

Fig. 13. Suppoſe the Part repreſented by A B, to be to that repreſented by B C, as *a* to *b*. Draw at pleaſure A *d*, and taking A *b* at pleaſure, make A *b* : *b c* : : *a* : *b*, and draw *b* B and *c* C meeting in O, and draw O V parallel to A *d*, cutting A B in V, which is the Vaniſhing Point ſought.

This is the reverſe of *Prop.* 13.

You may find V by Arithmetick, making C V = $\frac{a \times AC \times BC}{b \times AB - a \times BC}$.

PROP.

PROP. 26. PROB. 22.

[H]aving given the Representation of a given kind of Triangle, and its Vanishing Line; to find its Center and Distance.

Fig. 40. A B C is the given Representation of a Triangle, [an]d its Vanishing Line is F E, and consequently the Vanish[in]g Points of its Sides are F, D, and E.

Bisect FD and DE in G and H, and raise the Perpen[di]culars G I, H K, and make the Angle G I D equal to the [A]ngle that ought to be represented by F B D, and H K E [eq]ual to the Angle that ought to be represented by D A E, [an]d with the Centers I, and K, and Radius's I D and K D, [m]ake two Circles cutting each other in O. Then draw O P [cu]tting F E at right Angles in P, and P will be the Center, [an]d P O the Distance of the Vanishing Line F E.

DEMONSTRATION.

It is a known Property of the Circle, that the Angles [F]OD and DOE, are equal to G I D, and H K E; [t]herefore the Angles F B D, or A B C, and B A C, repre[se]nt Angles equal to G I D, and H K E, by *Prop.* 12.

PROP.

Prop. 27. Prob. 23.

Having given the Representation of a given kind [of] Parallelogram; to find its Vanishing Line, Cent[er] and Distance.

Fig. 41. ABCD is the given Representation of a Par[al]lelogram. Continue the opposite Sides, till they meet [in] E and F, then will EF be the Vanishing Line. Dr[aw] the Diagonal AC, and by means of the Triangle AB[C] find the Center and Distance by the foregoing *Proposition.*

DEMONSTRATION.

For the Original Figure being a Parallelogram, the opp[o]site Sides are parallel; wherefore their Representations m[ust] meet in their Vanishing Points, and consequently EF will [be] the Vanishing Line.

Prop. 28. Prob. 24.

Having given the Representation ABCD (Fig. 42[)] *of a given kind of Trapezium, to find its Vanish[]ing Line, Center and Distance.*

Draw the Diagonals AC and DB meeting in E, and [by] the given Proportion of the Originals of AE, EC, B[E,] ED, find the Vanishing Points F and G of the Lines A[C] and DB (by *Prop.* 25.) and you will have the Vanishi[ng] Line GF. Then by the given Species of the Original of t[he] Triangle ABE, find the Center and Distance (by *Prop.* 2[6]

Pro[p.]

Prop. 29. Prob. 25.

Having given the Representation of a right-angled Parallelepipedon; to find the Center and Distance of the Picture, and the Species of the Original Figure.

Fig. 43. A B C D E F G is the Representation given. Continue the Sides of the Figure till they meet in their Vanishing points H, I, K, and you will have the Vanishing Lines H I, I K, K H of all the Faces of the Figure. Then from H and I draw Perpendiculars to I K and H K, meeting in S, which will be the Center of the Picture, L and M being the Centers of the Vanishing Lines I K and H K. Upon the Diameter I K make a Circle cutting H L in O. Then will L O be the Distance of the Vanishing Line I K, Whence the Distance of the Picture is easily found. Draw the Diagonal G F cutting I K in its Vanishing Point N, and draw O N. Then will N O K be the Angle represented by A G F, and N O I will be the Angle represented by F G E; whence you have the Species of the Face A F E G. And in the same manner you may find the Species of the other Faces.

DEMONSTRATION.

The opposite Sides of the Figure being to represent Parallels, the Points where they meet must be their Vanishing Points, and consequently the Lines H I, I K, K H will be their Vanishing Lines. But because of all the Angles being Right-angles, H and I will be the Vanishing Points of Lines perpendicular to the Planes whose Vanishing Lines are I K, and H K, and consequently L and M will be the Centers of those Vanishing Lines, and S the Center of the Picture, as is sufficiently evident by *Prop.* 8. But because of the Circle, the Angle I O K is a Right-angle. Wherefore the Distance L O is rightly found to make E G A represent a Right-angle; whence the Angles N O K, and N O I are found, which are represented by F G A, and F G E, by *Prop.* 12.

When the Sides A B, G C, E D are parallel to each othe (*n* : 2.) the Point H will be infinitely Distant, and the Va nishing Line I K will pass thro' the Center of the Pictu the Points L and S in this case co-inciding. But beca the Point L in this Case is left undetermined, the Species of t Figure is also undetermined. So that the Point O may taken any where in the Semicircle I O K. Whence if t Semicircle I O K be placed perpendicular to the Picture, Spectator's Eye placed any where in that Circumference, w take the Figure A D for a Right-angled Parallelepipedon; t the Species of it will be different, according to the differe Places of the Spectator's Eye. But this is, provided the Li AB, G C, &c. be perpendicular to the Vanishing Line K otherwise the Figure can't represent a Right-angled Paralle pipedon, but one inclined on a Right-angled Base.

I leave it to the Artists to consider whether this Observati may not be of Use in Painting the Scenes of a Theater. F hence it appears, that if I and K were the two Points, that the common Books of Perspective are call'd the Points of D stance, and the Buildings in the Scene were so drawn, th their Sides should run to those Points, (as in the present Figure they would always appear Right-angled to the Spectators in the front Seats of the Boxes, their Eyes being all in the Cir cumference of the Semicircle I O K; for tho' the Proportio of their Sides were something alter'd, that would be no gre Inconvenience.

FINIS.

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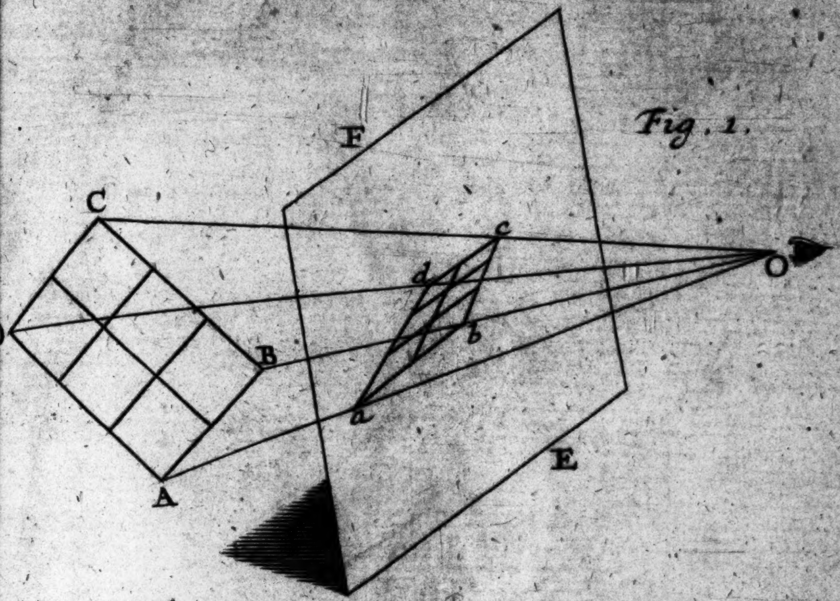

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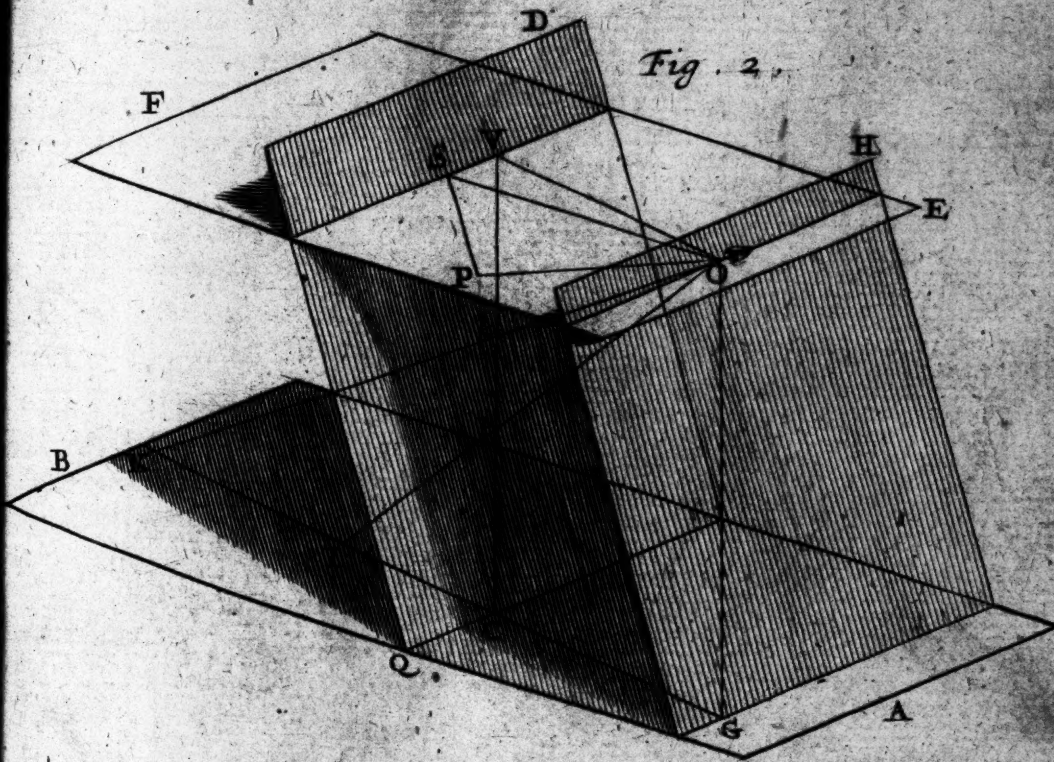

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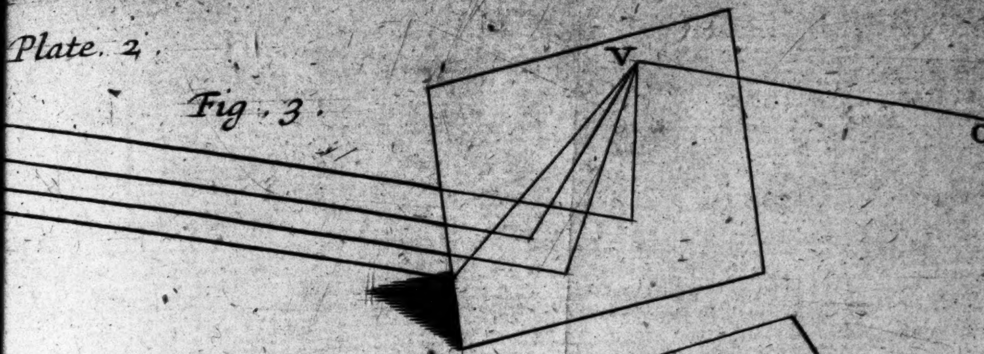

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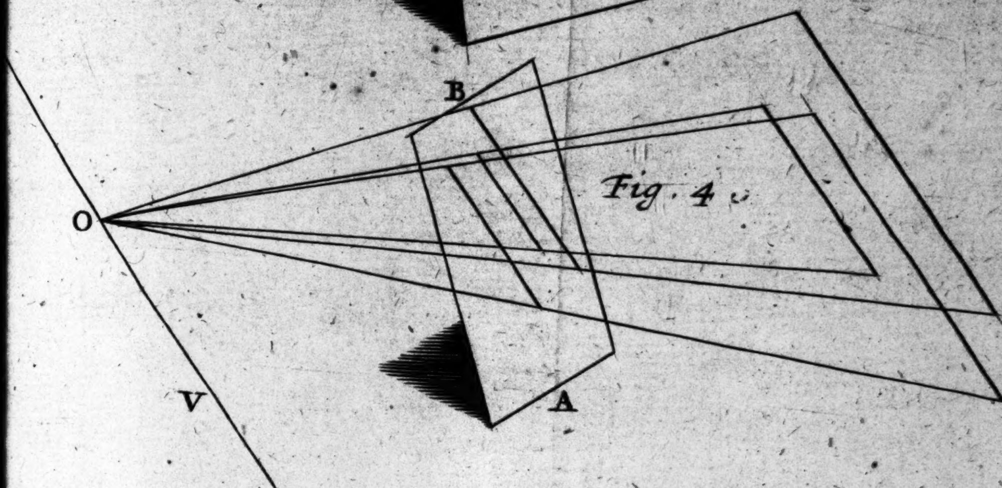

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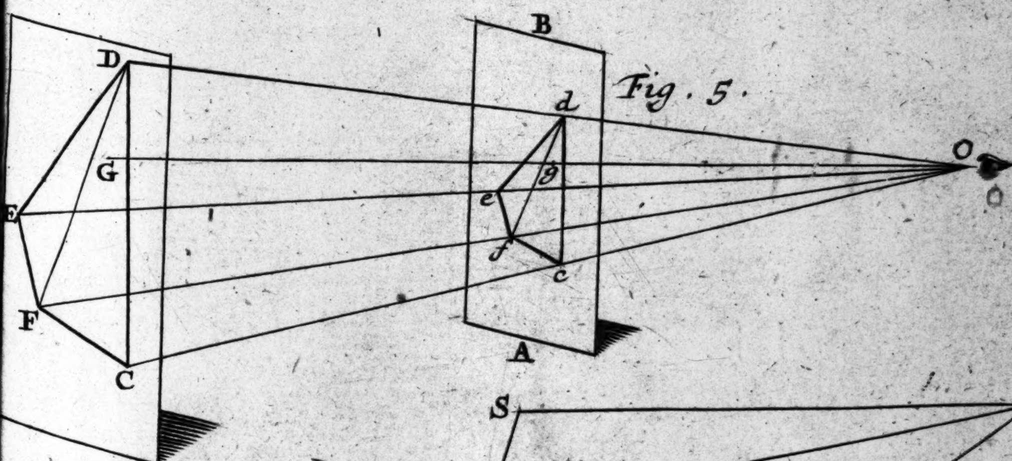

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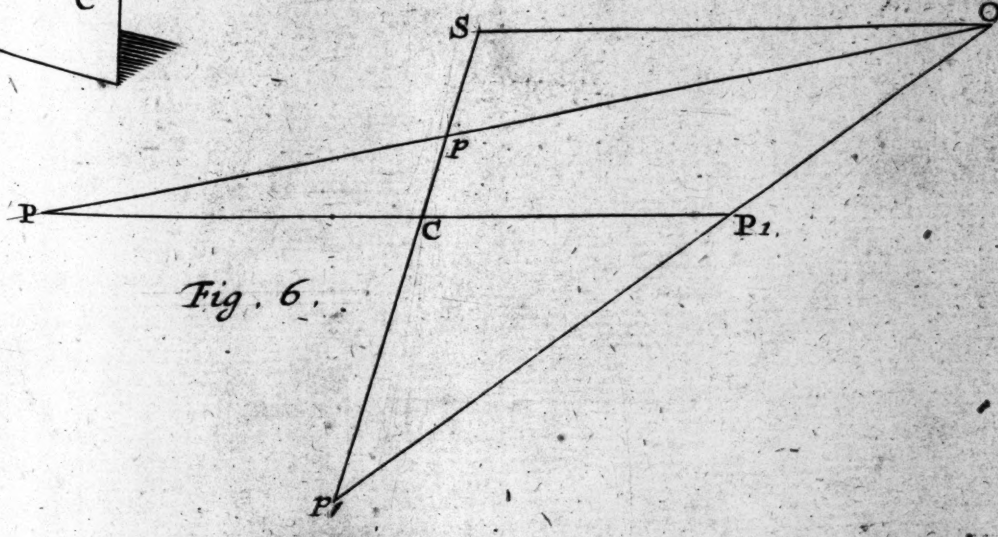

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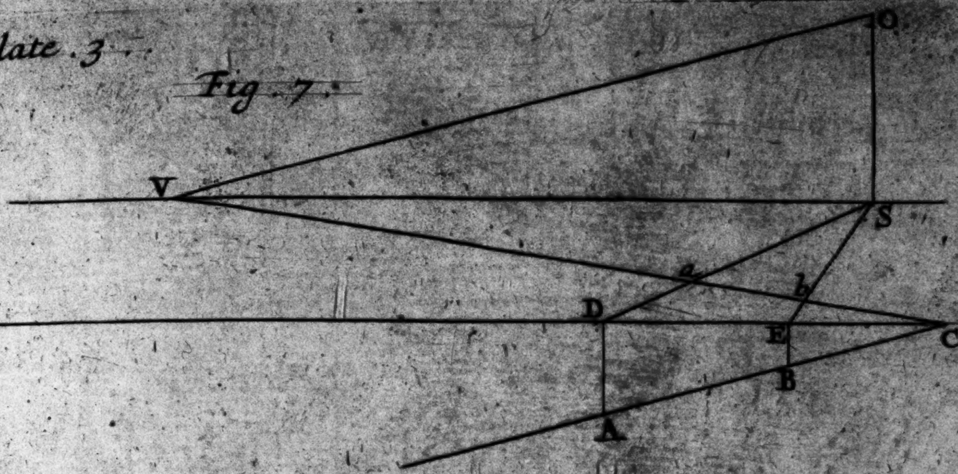

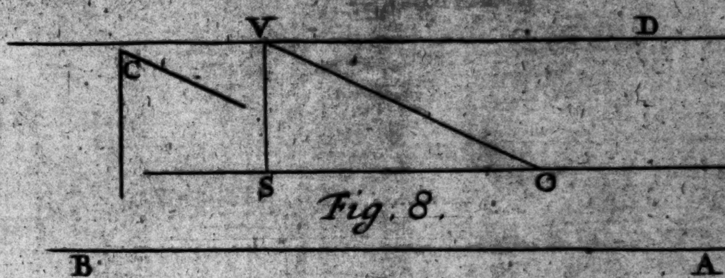

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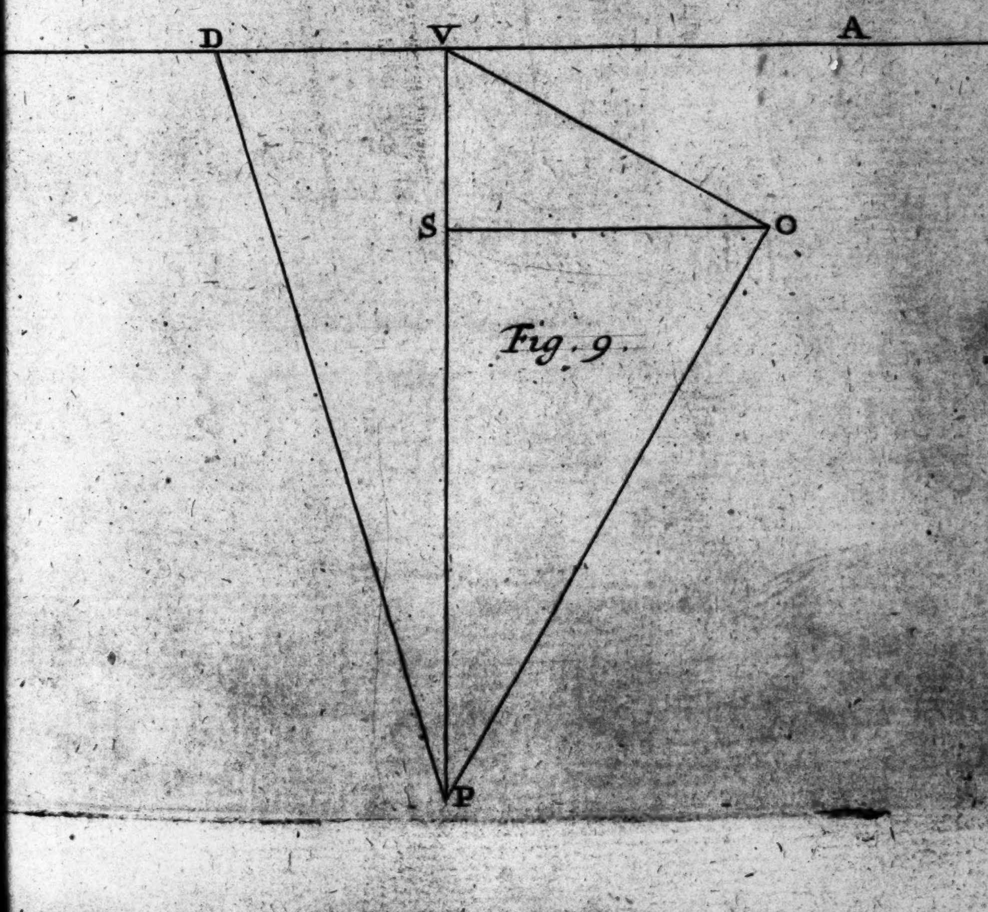

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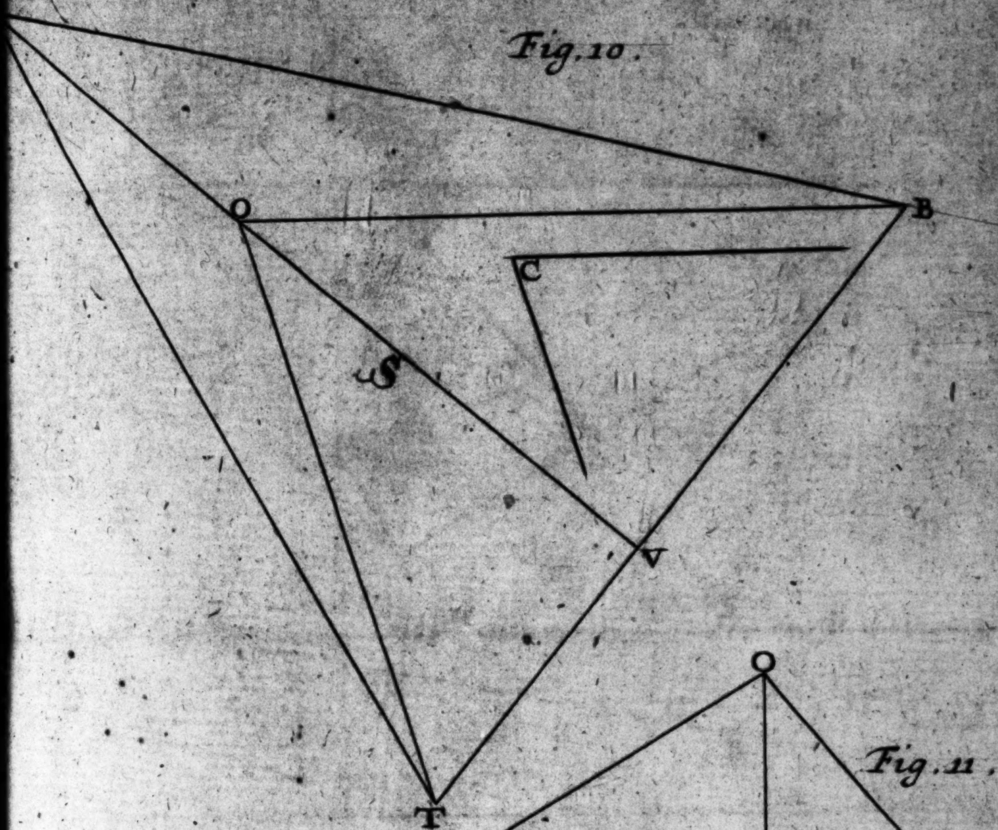

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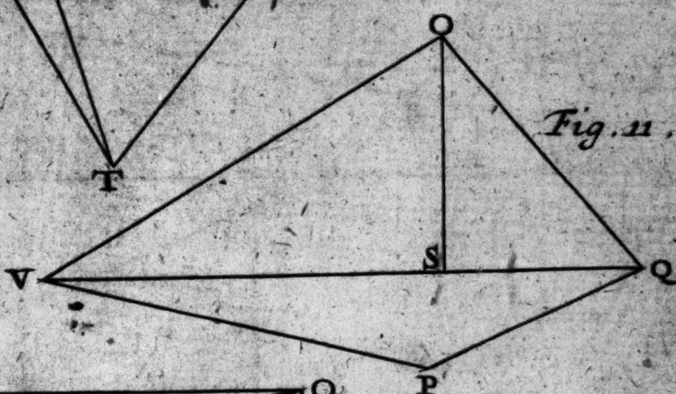

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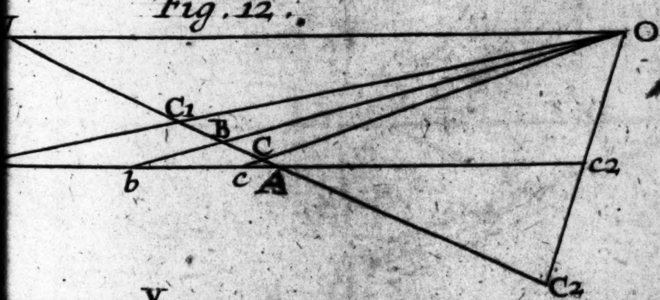

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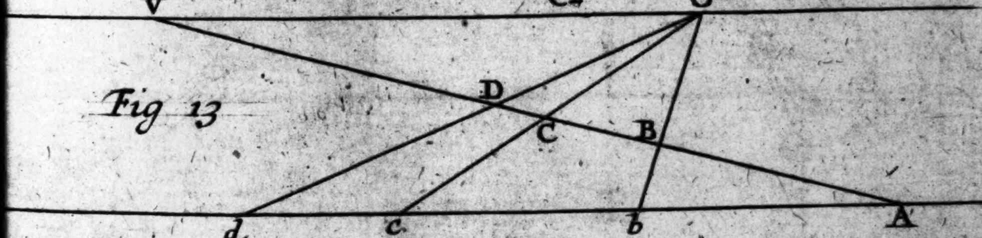

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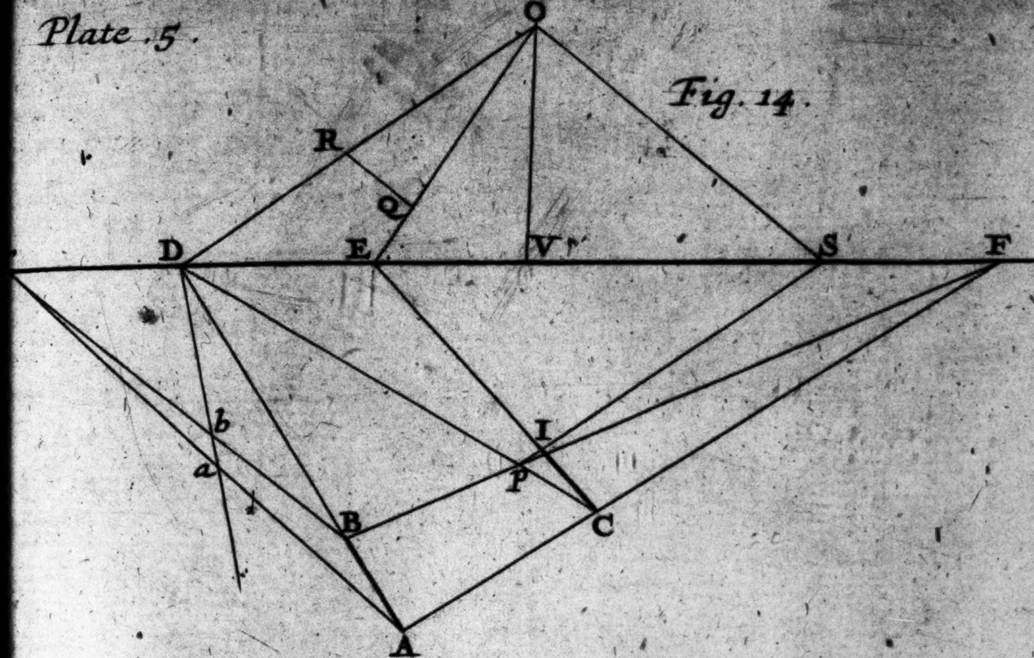

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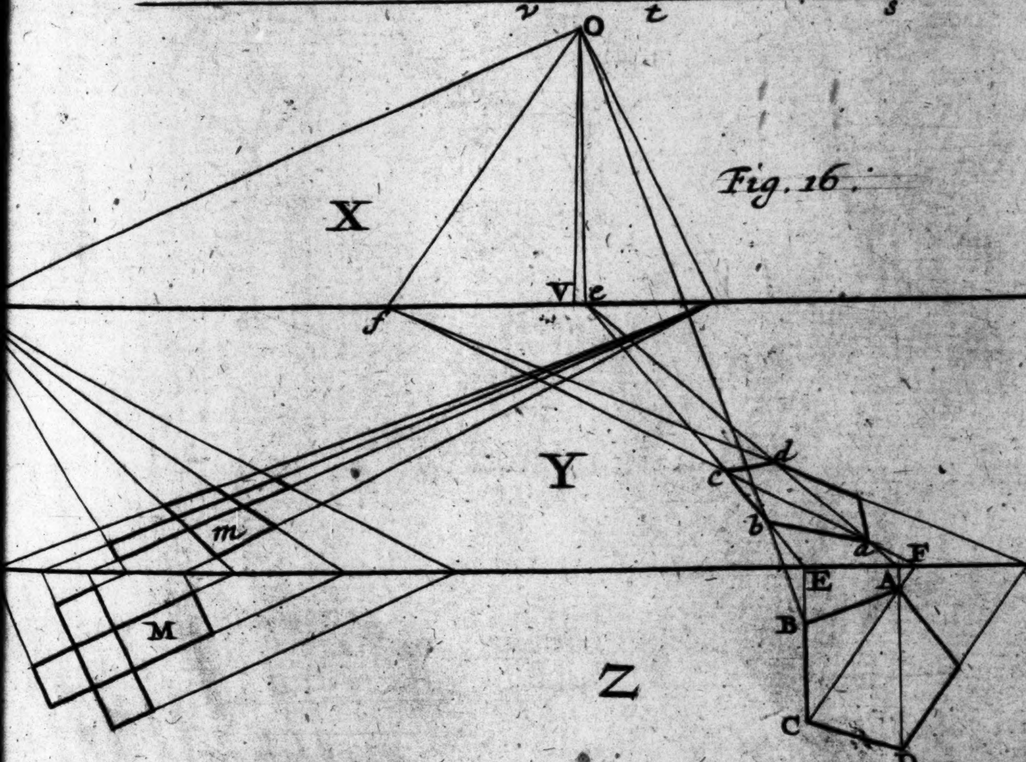

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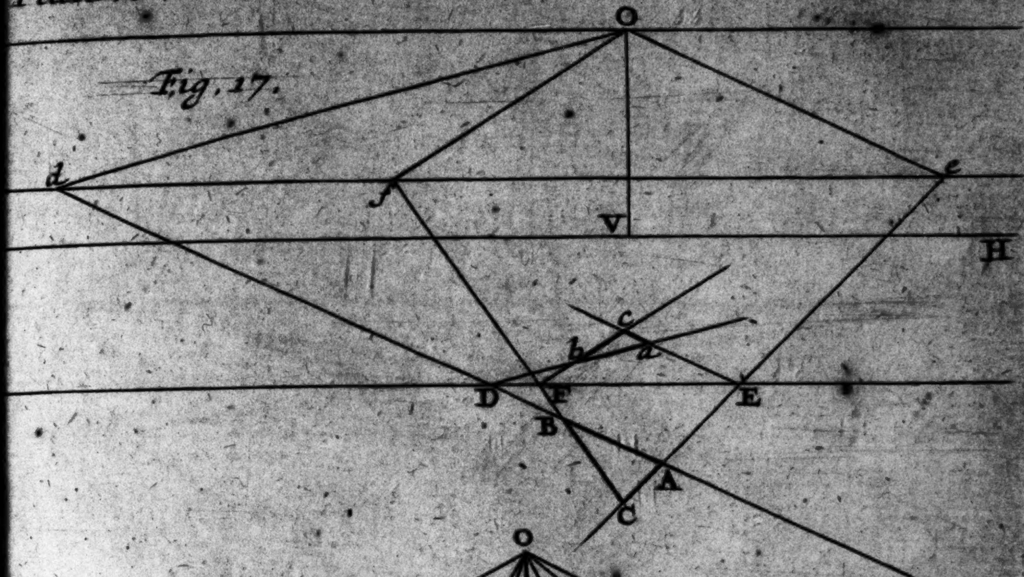

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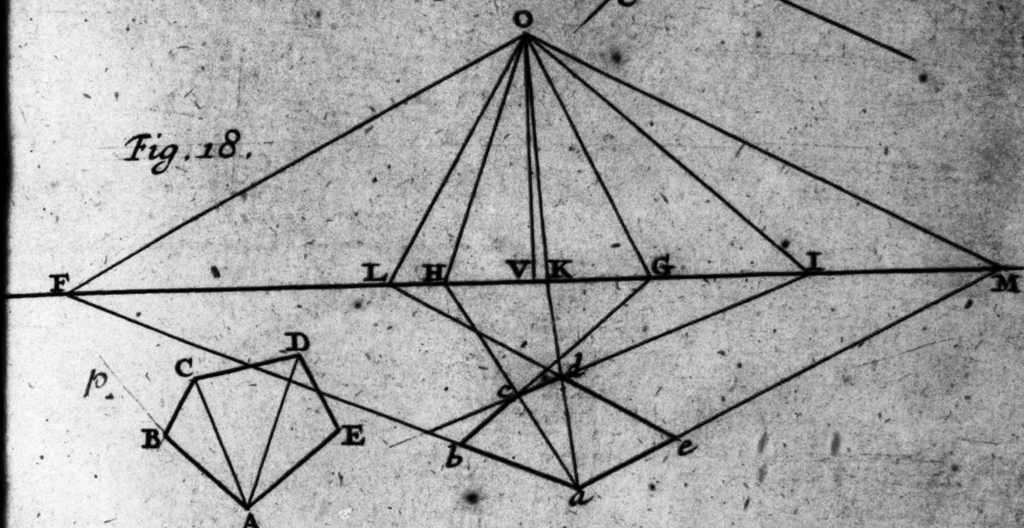

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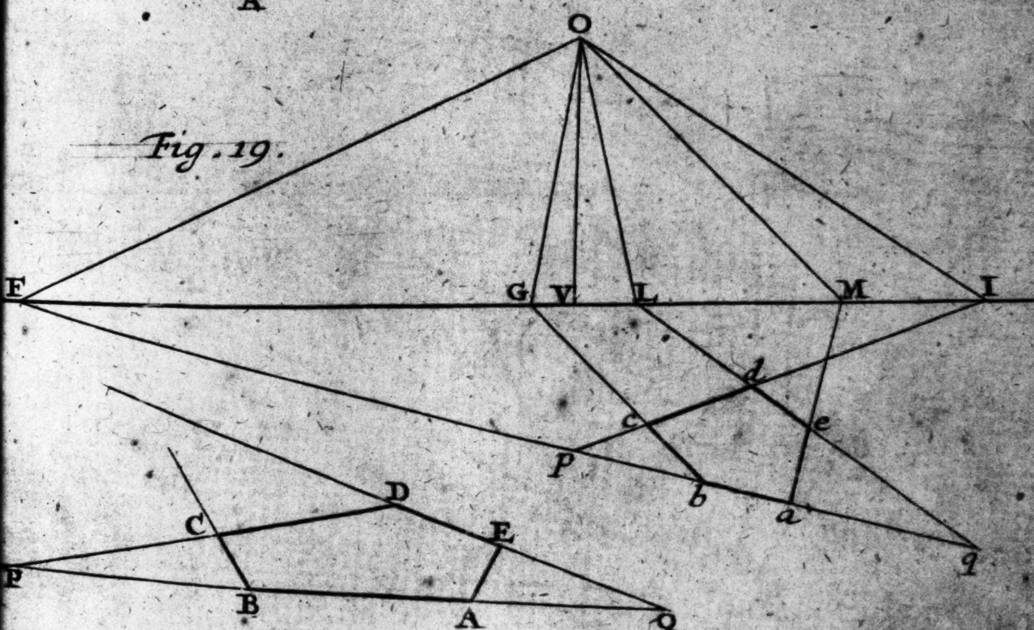

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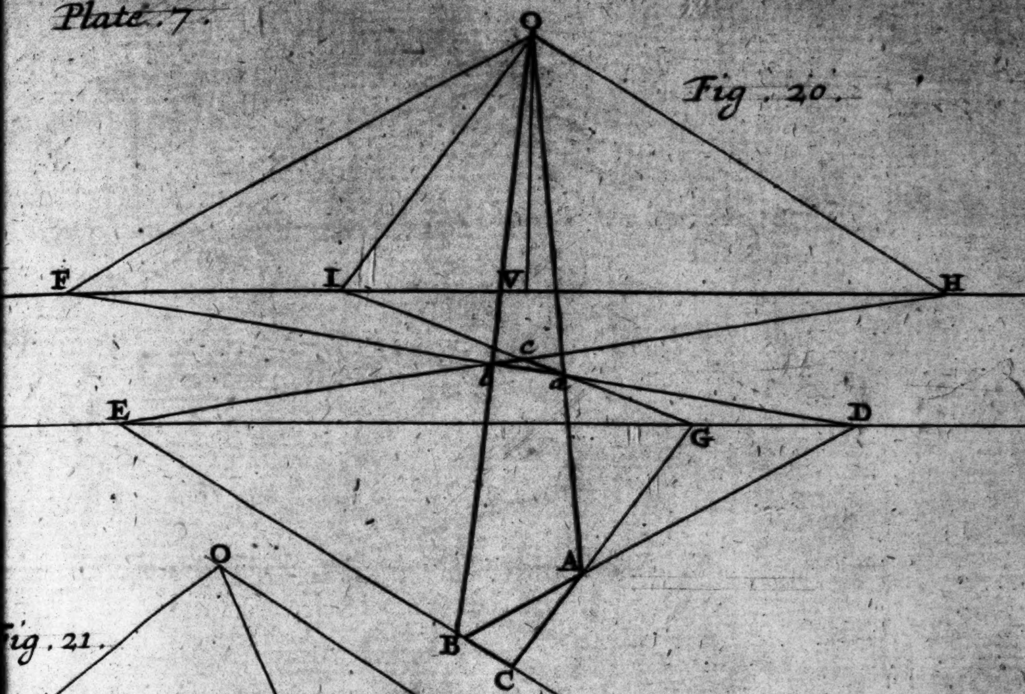

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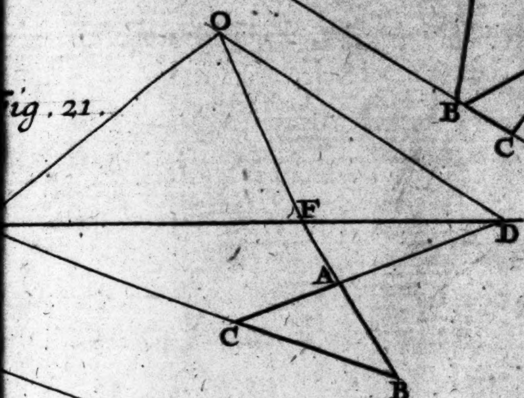

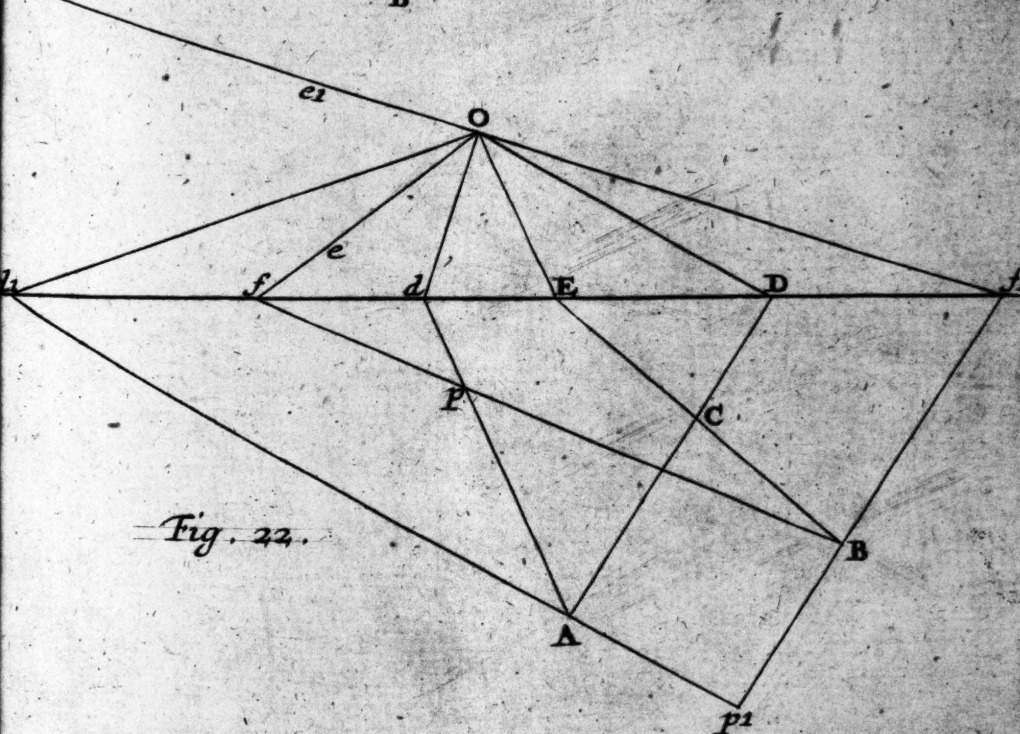

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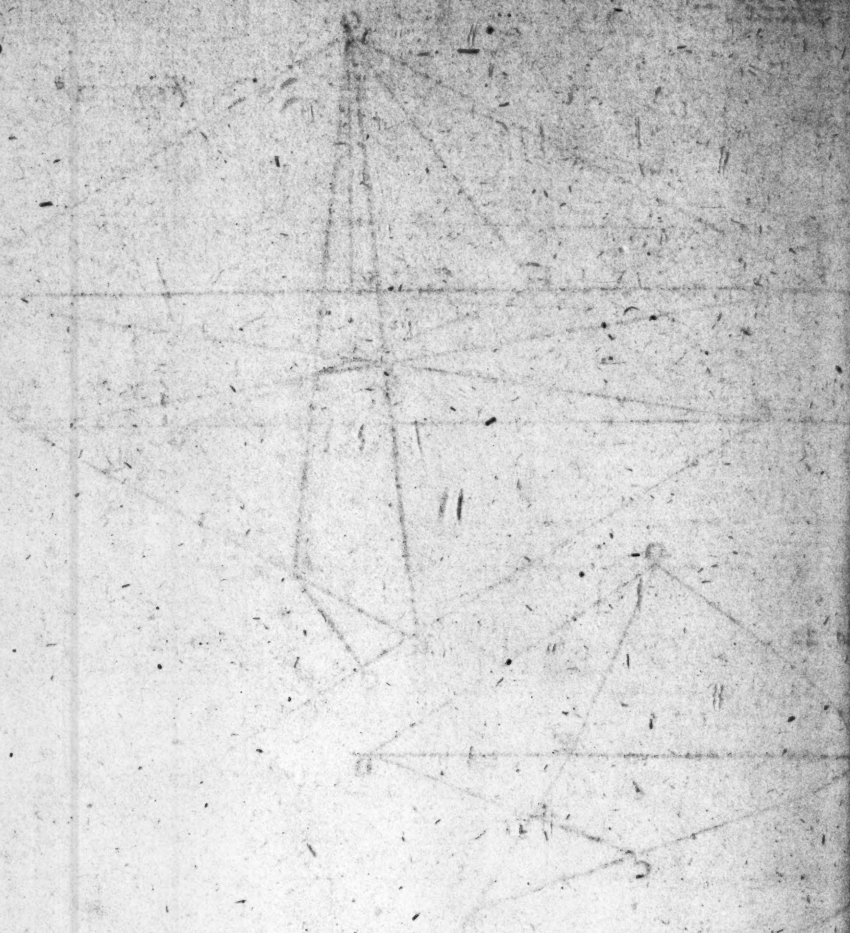

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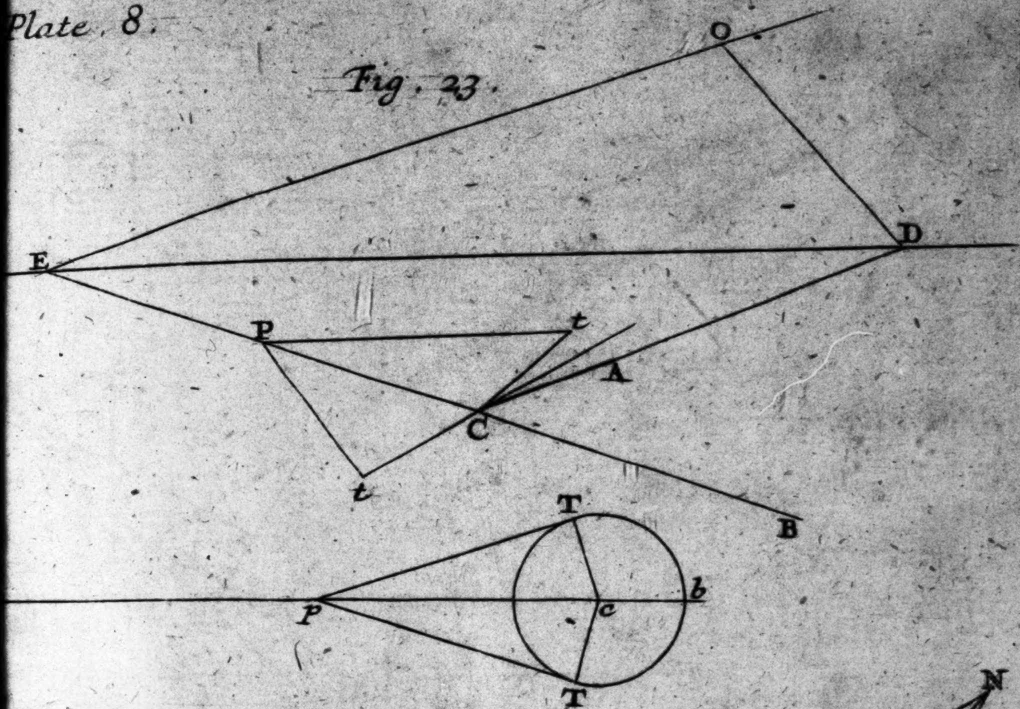

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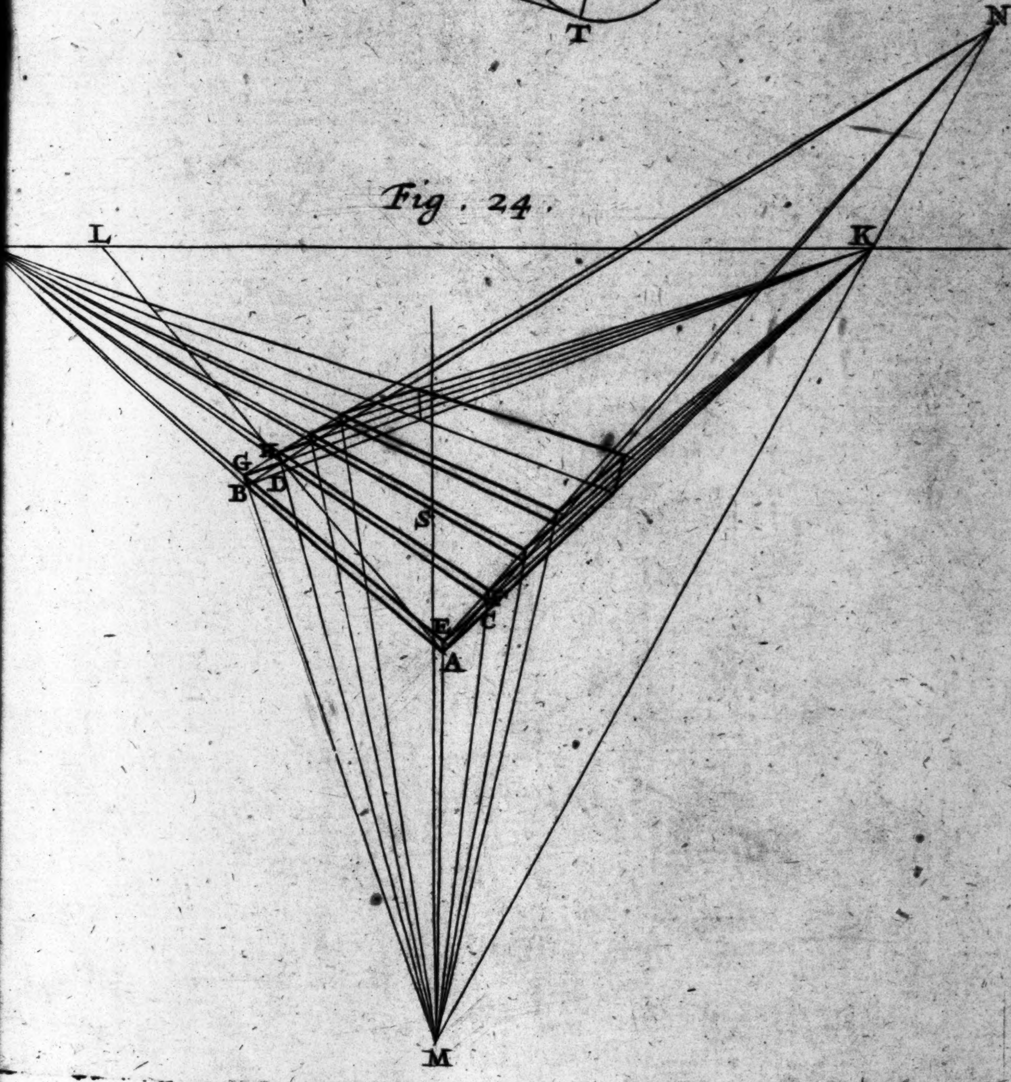

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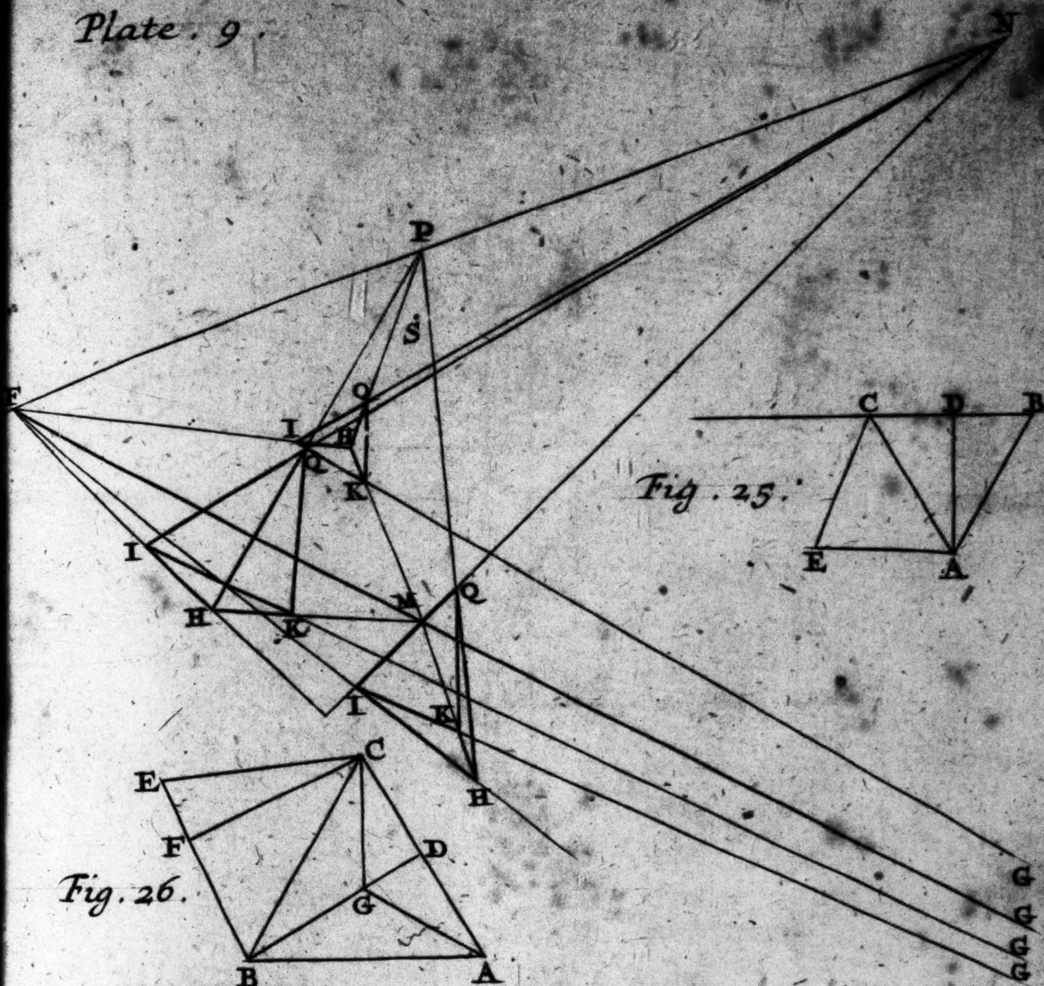

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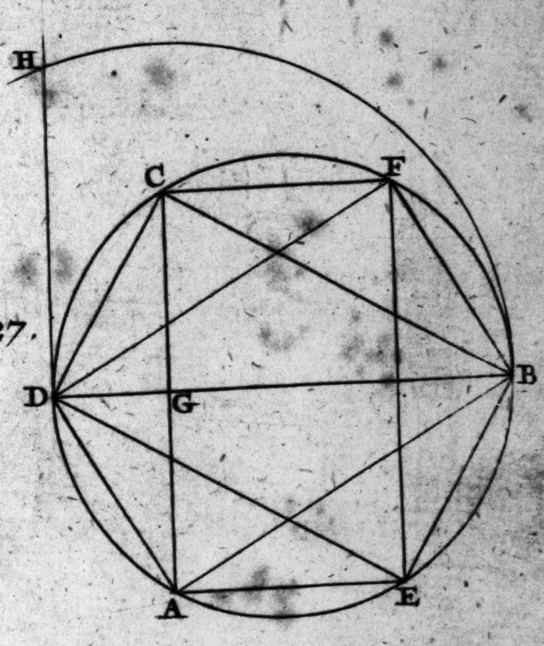

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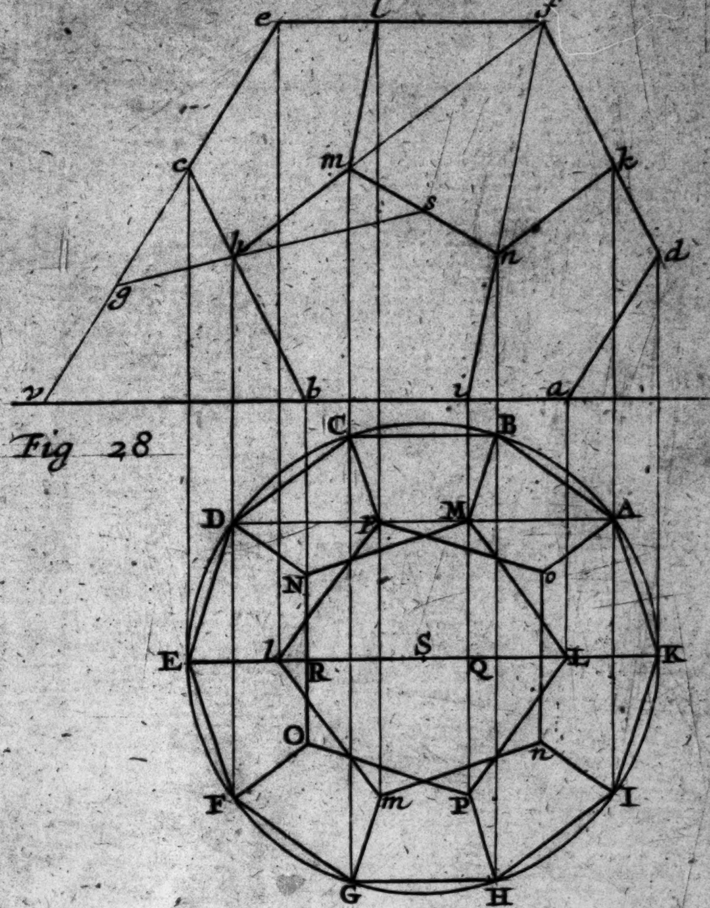

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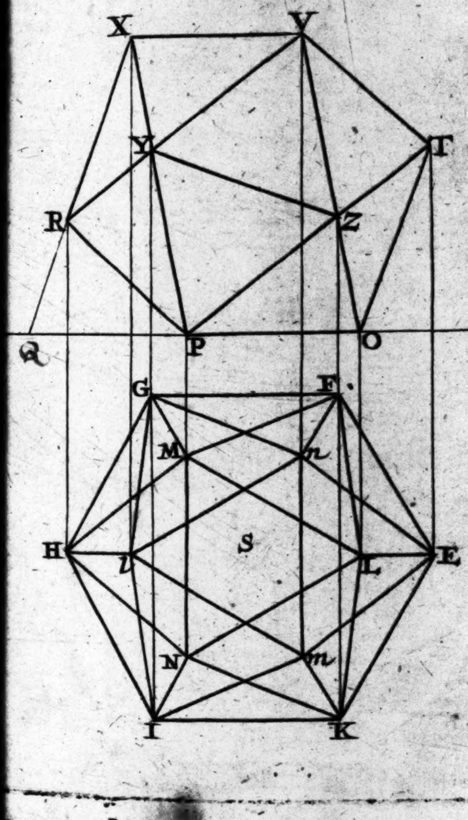

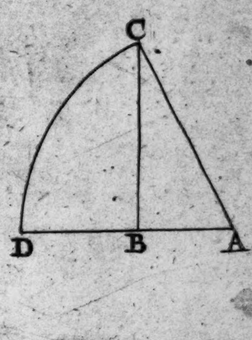

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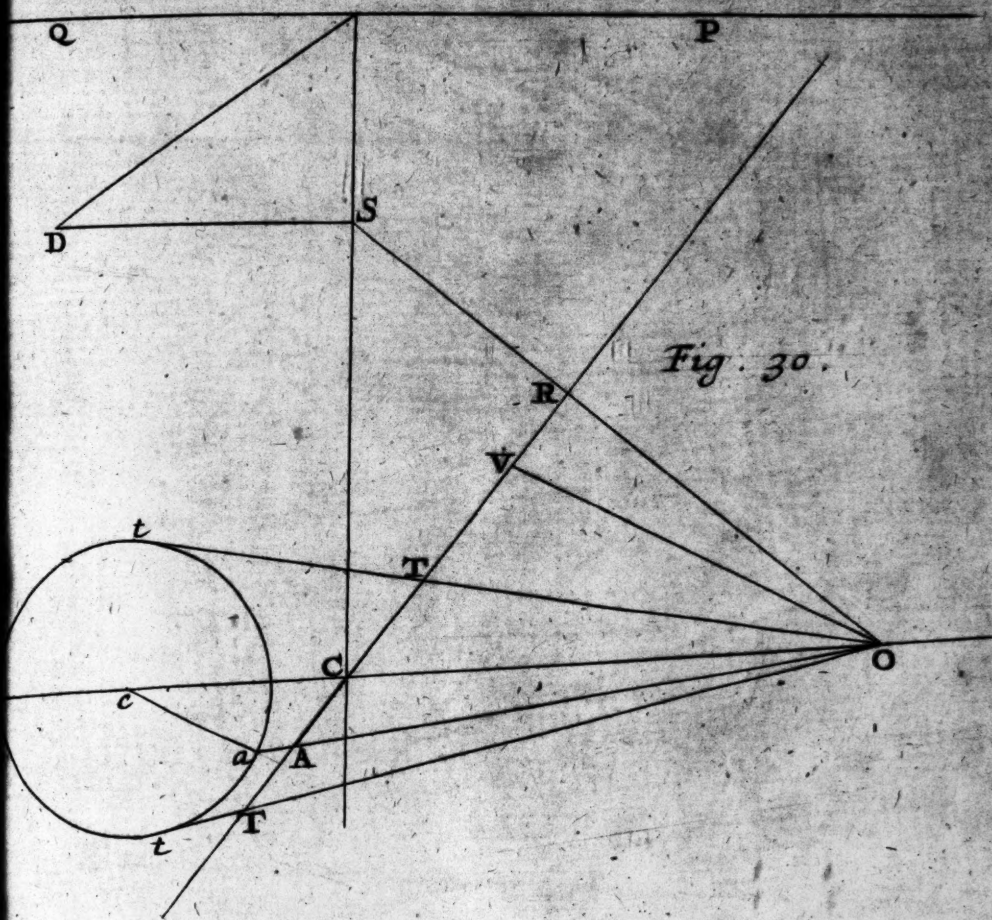

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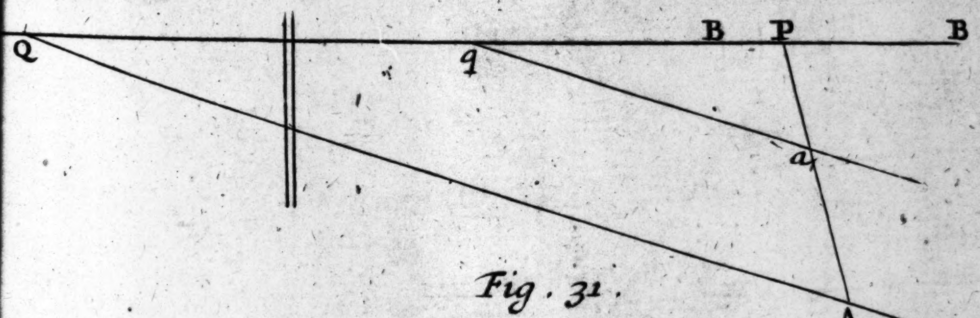

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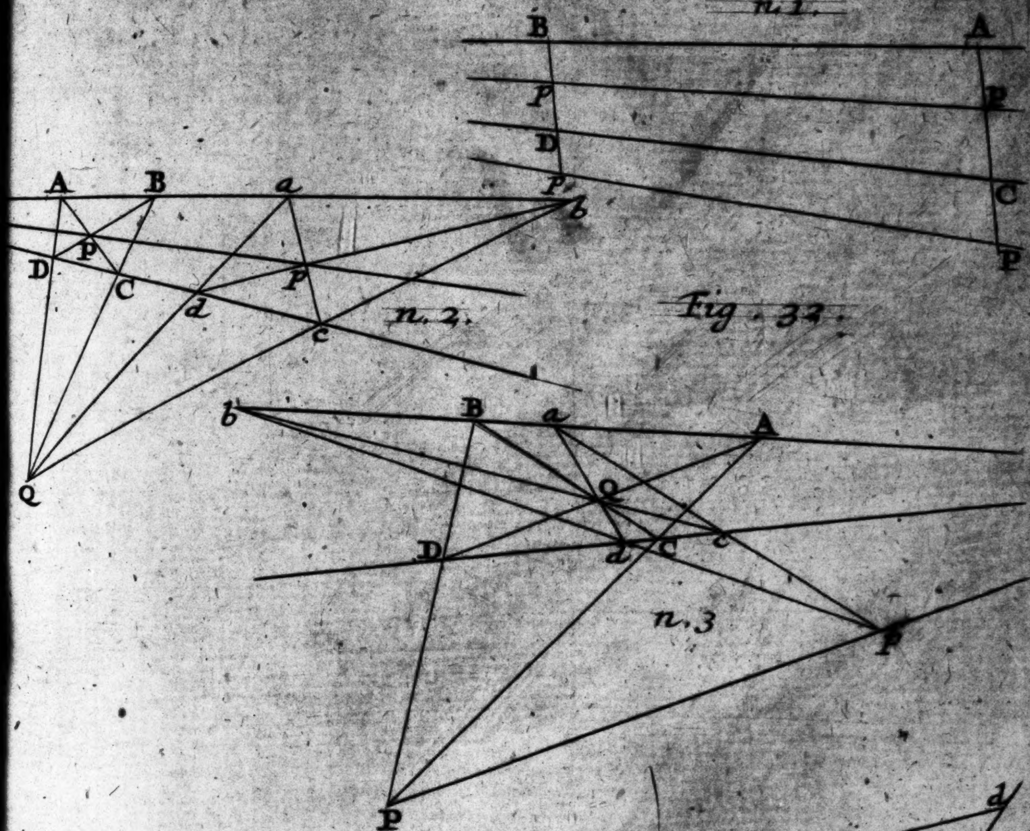

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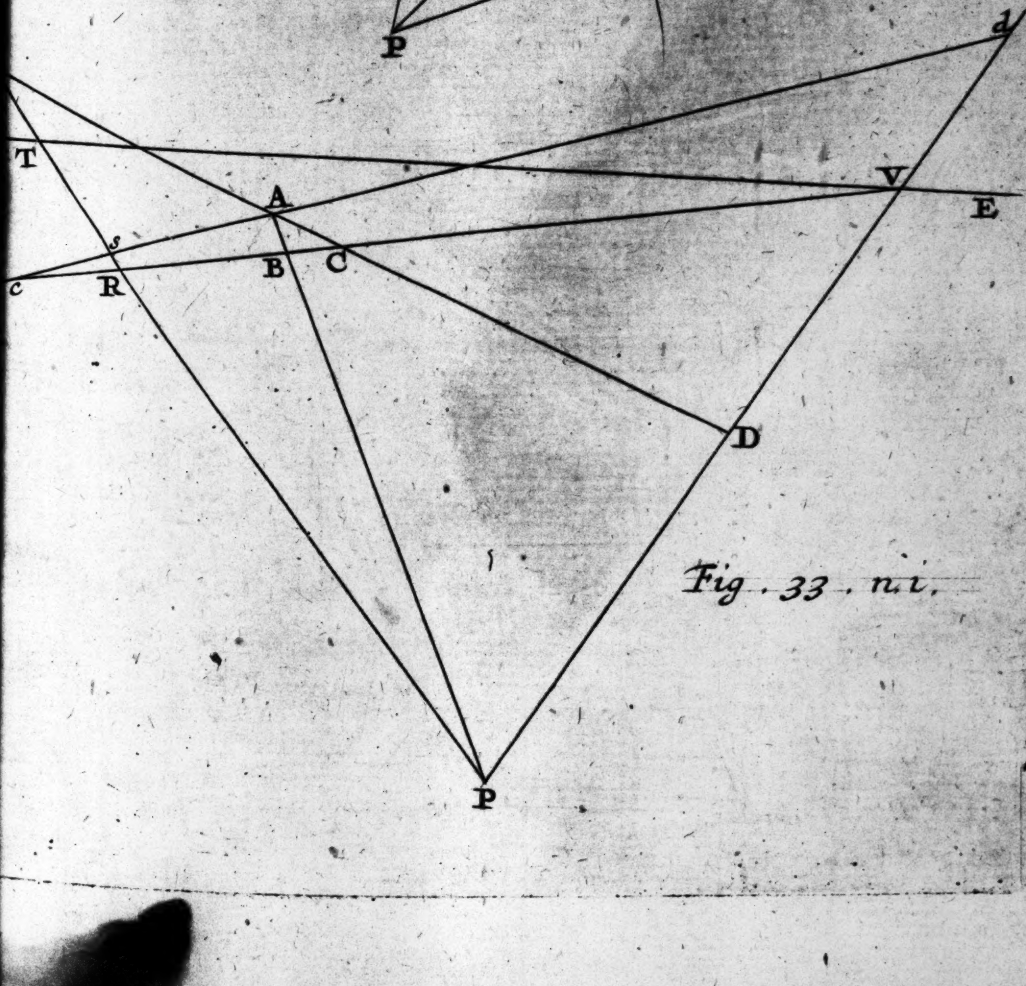

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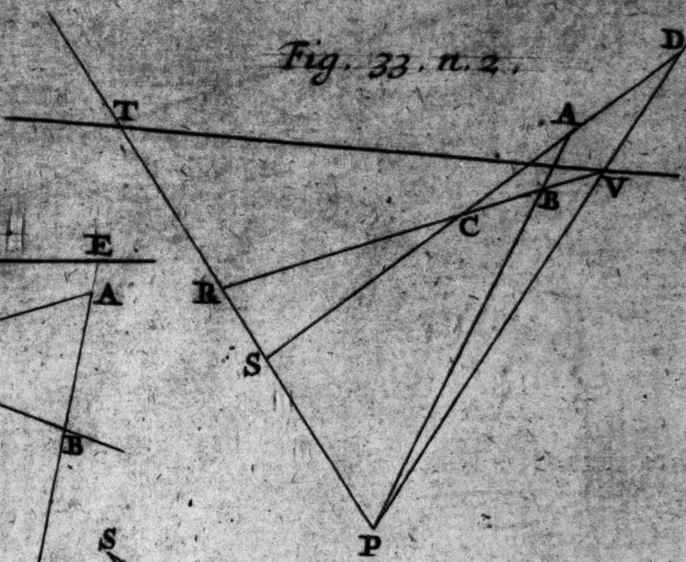

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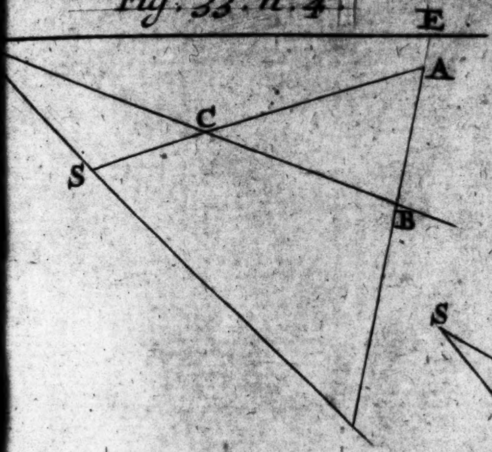

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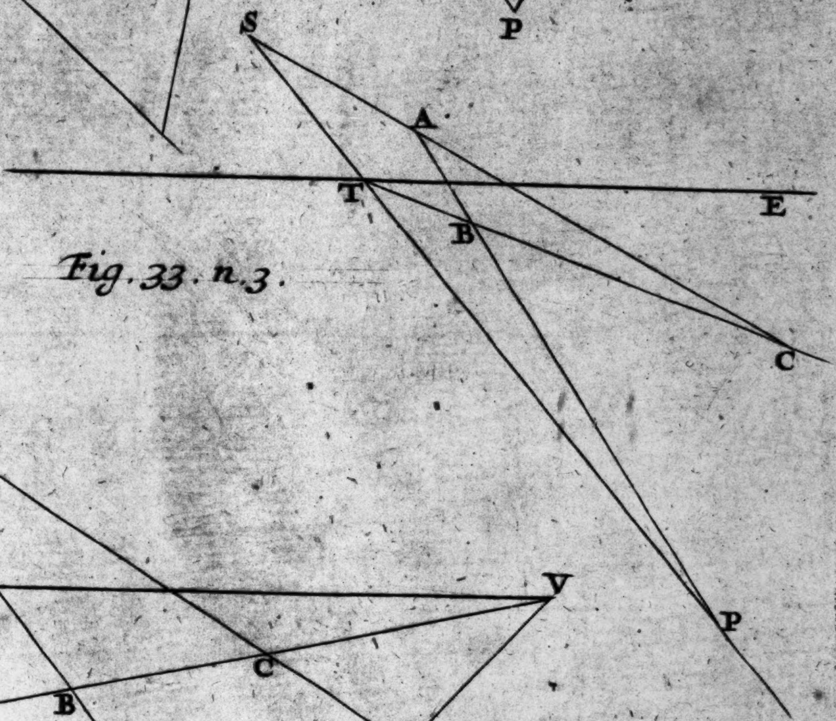

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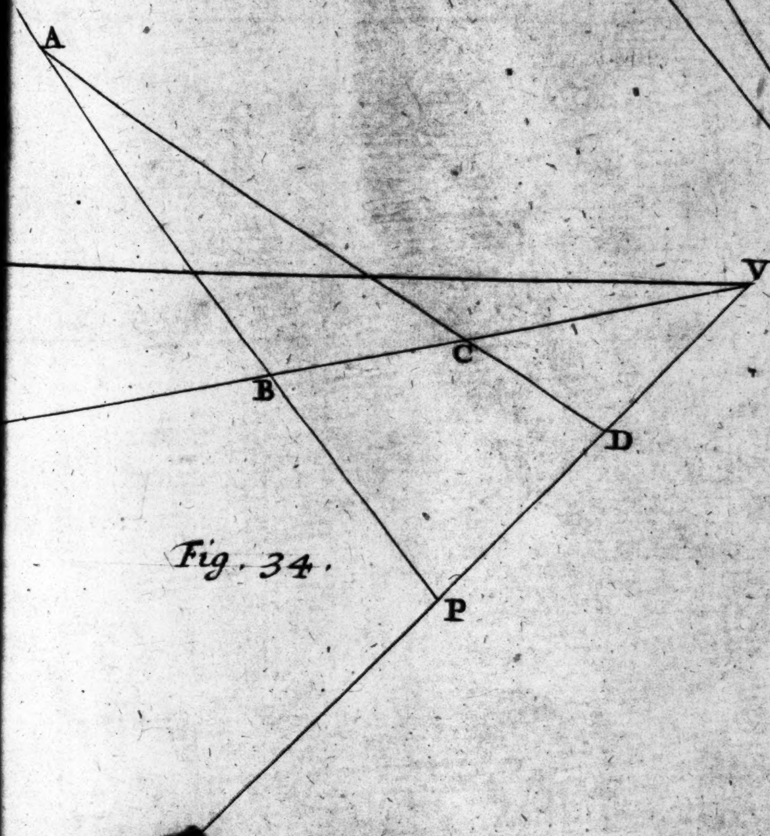
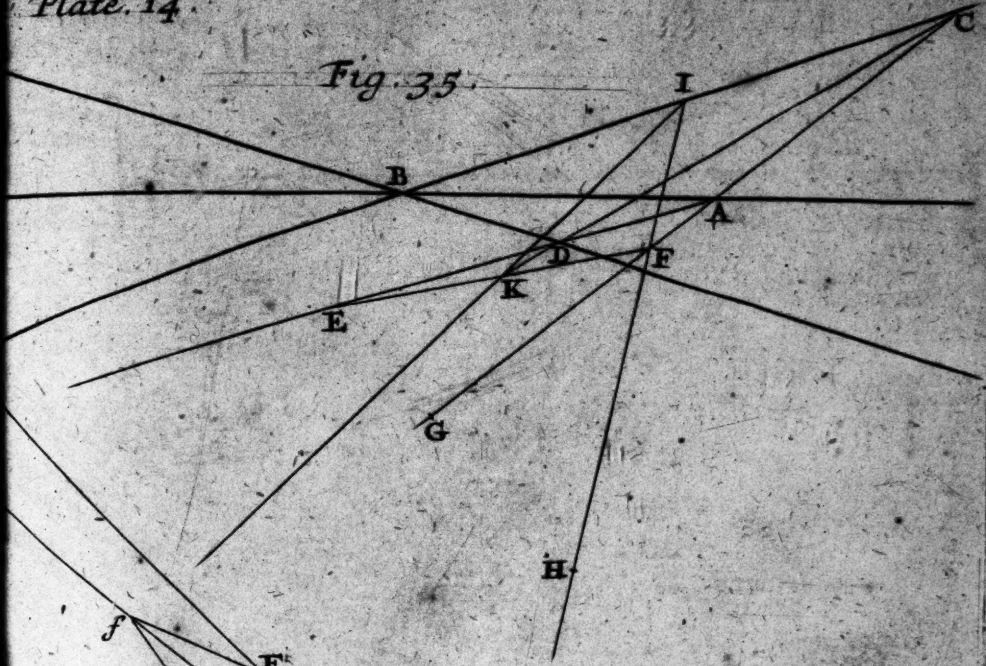


Fig. 34.

Plate. 14



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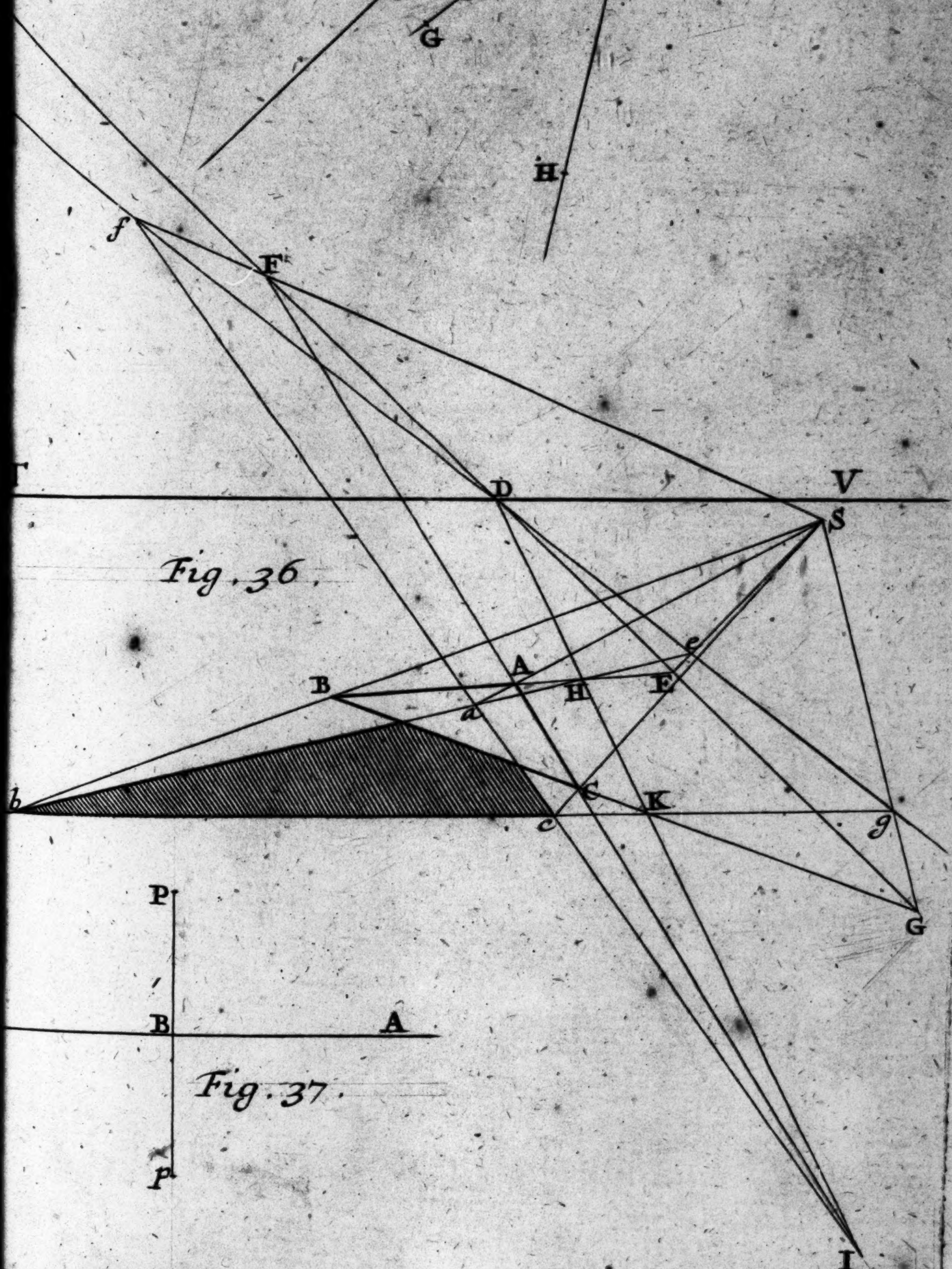

Fig. 36.

Fig. 37.

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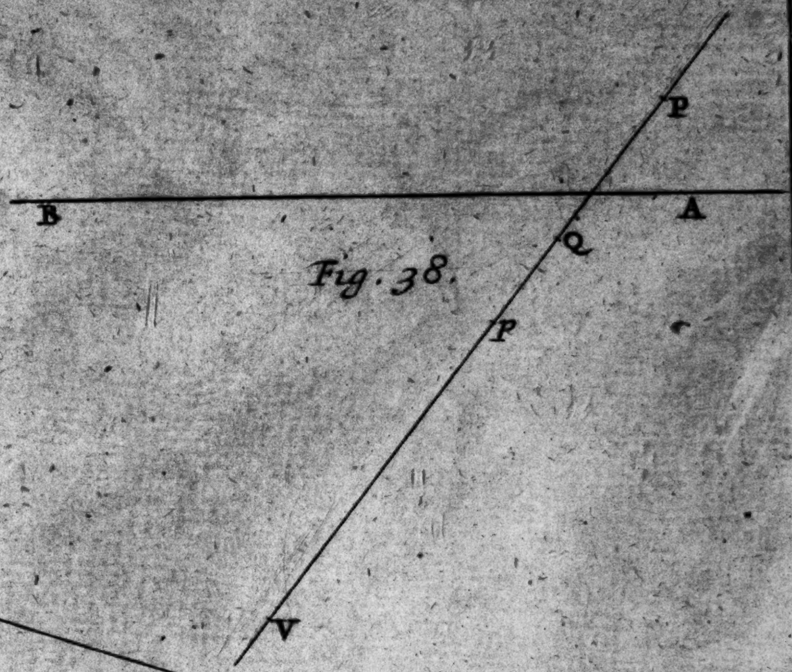

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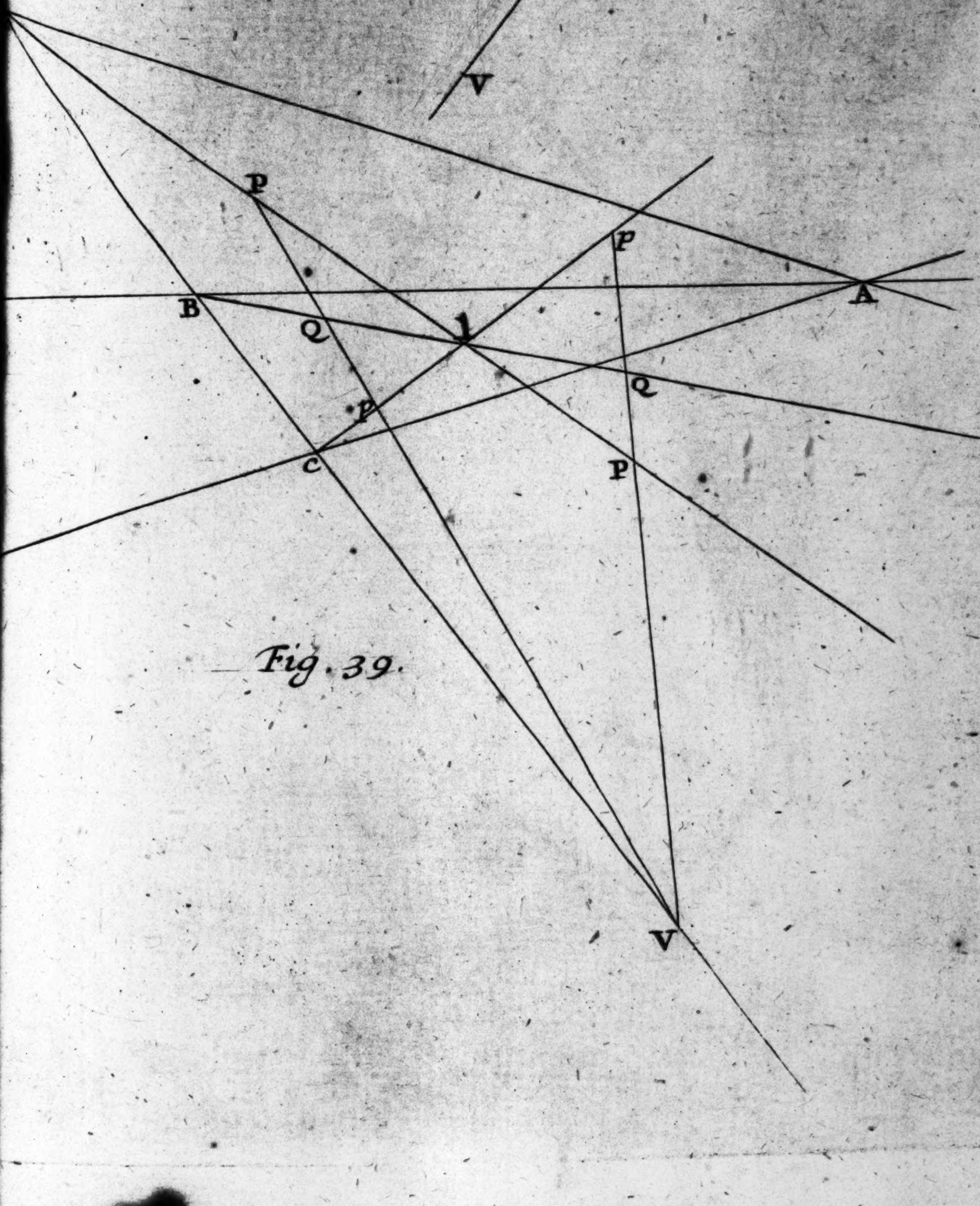

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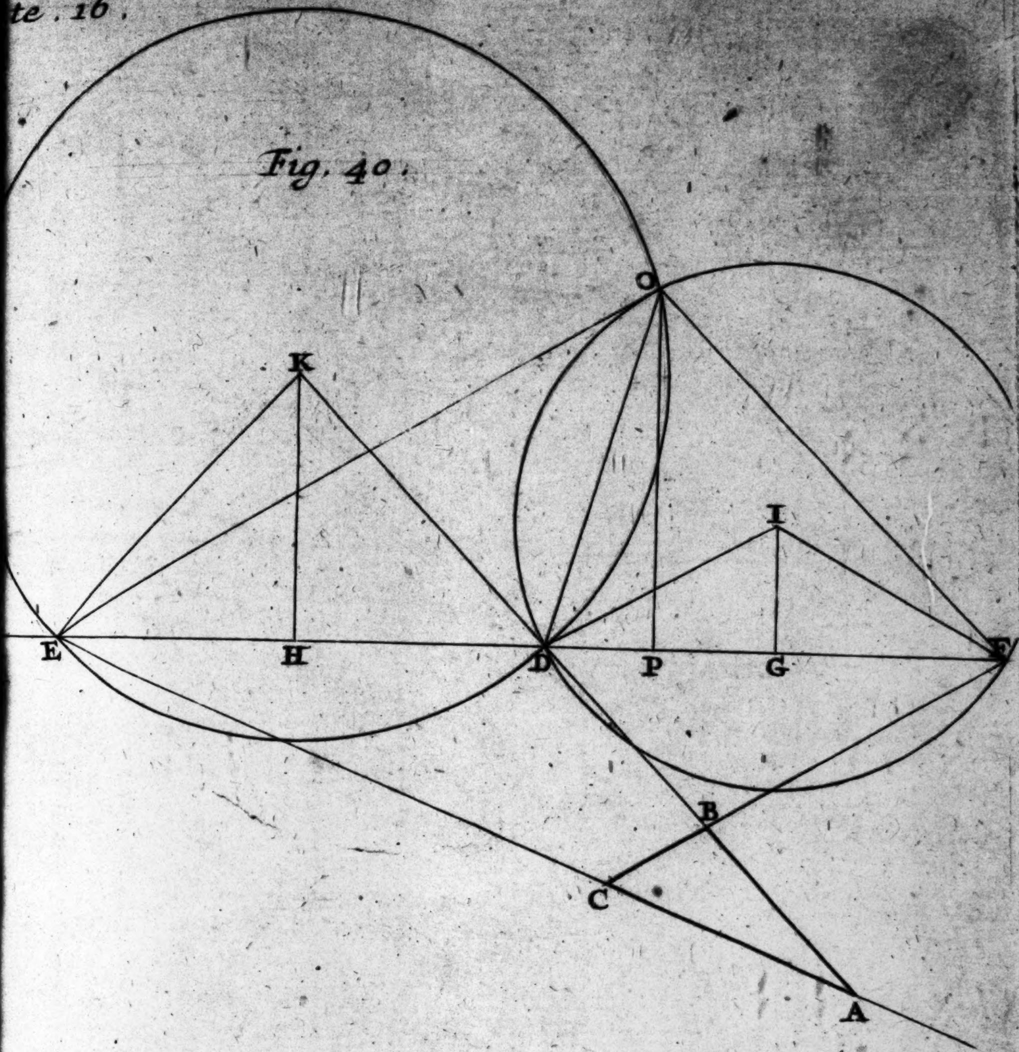

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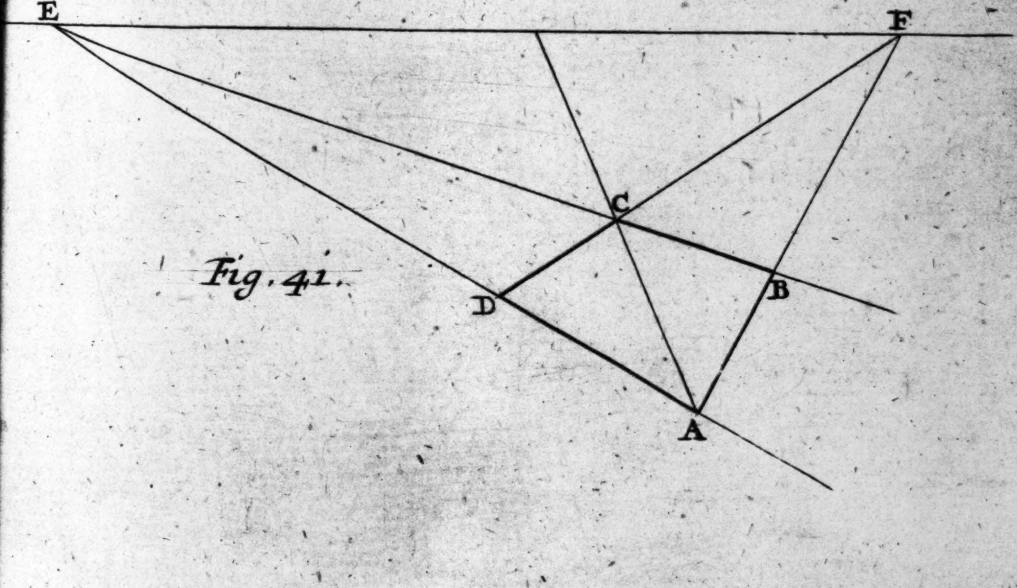

Fig. 41.

26, 03 : 6
18, 15 : 9

44, 19 : 3

32, 13 : 4

Plate. 17.

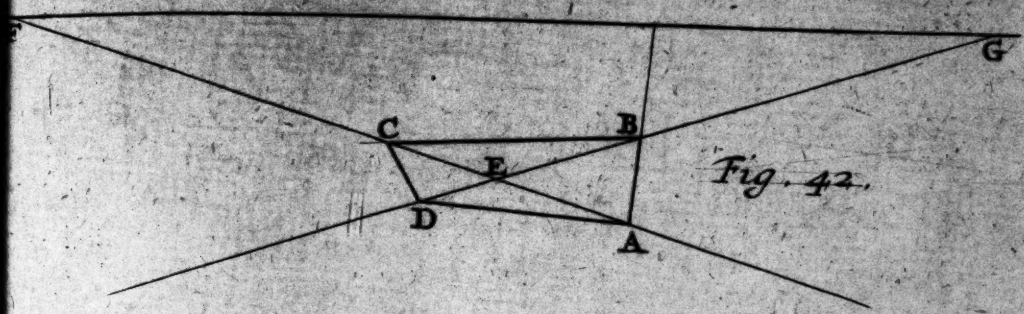

Fig. 42.

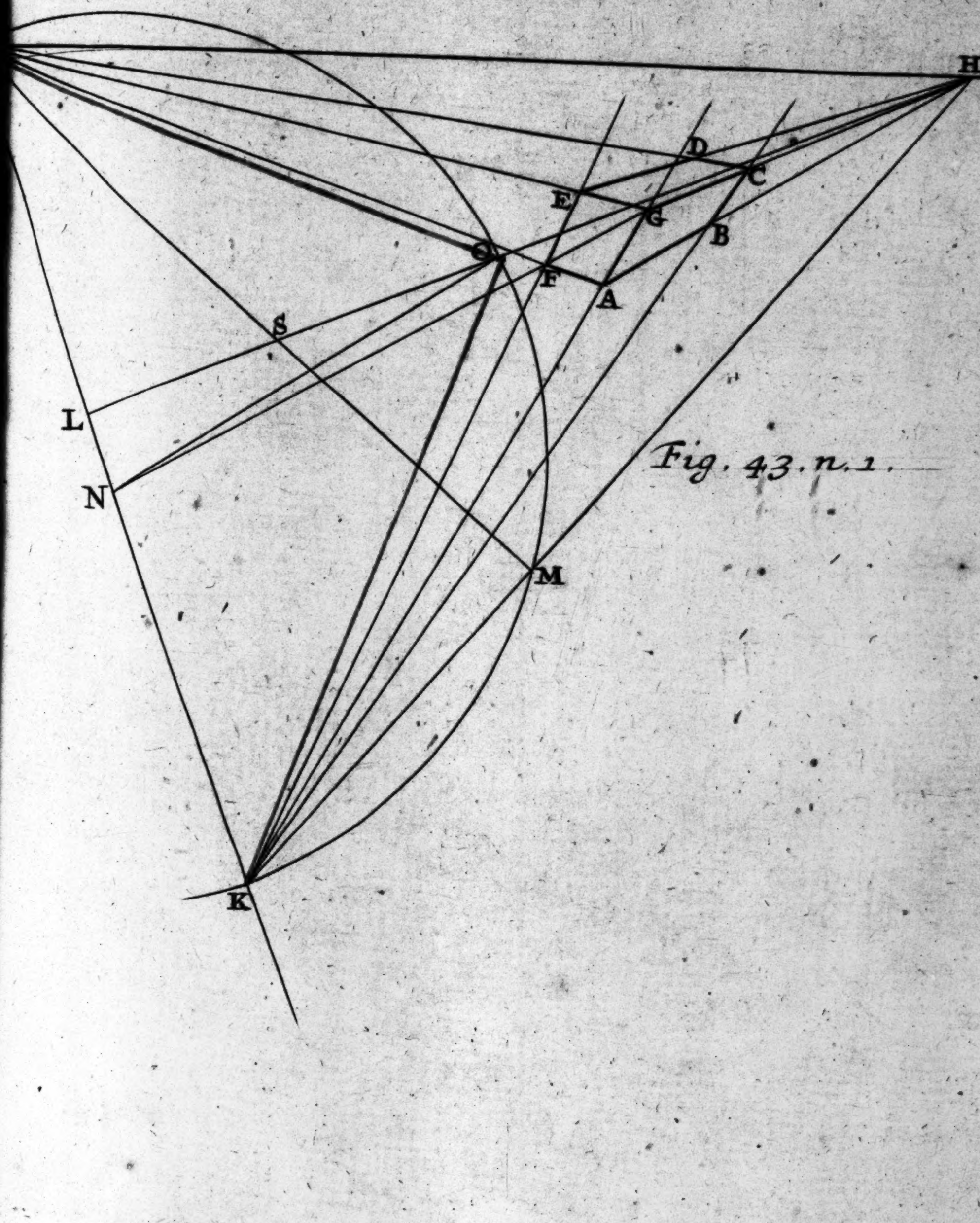

Fig. 43. n. 1.

Plate . 18.

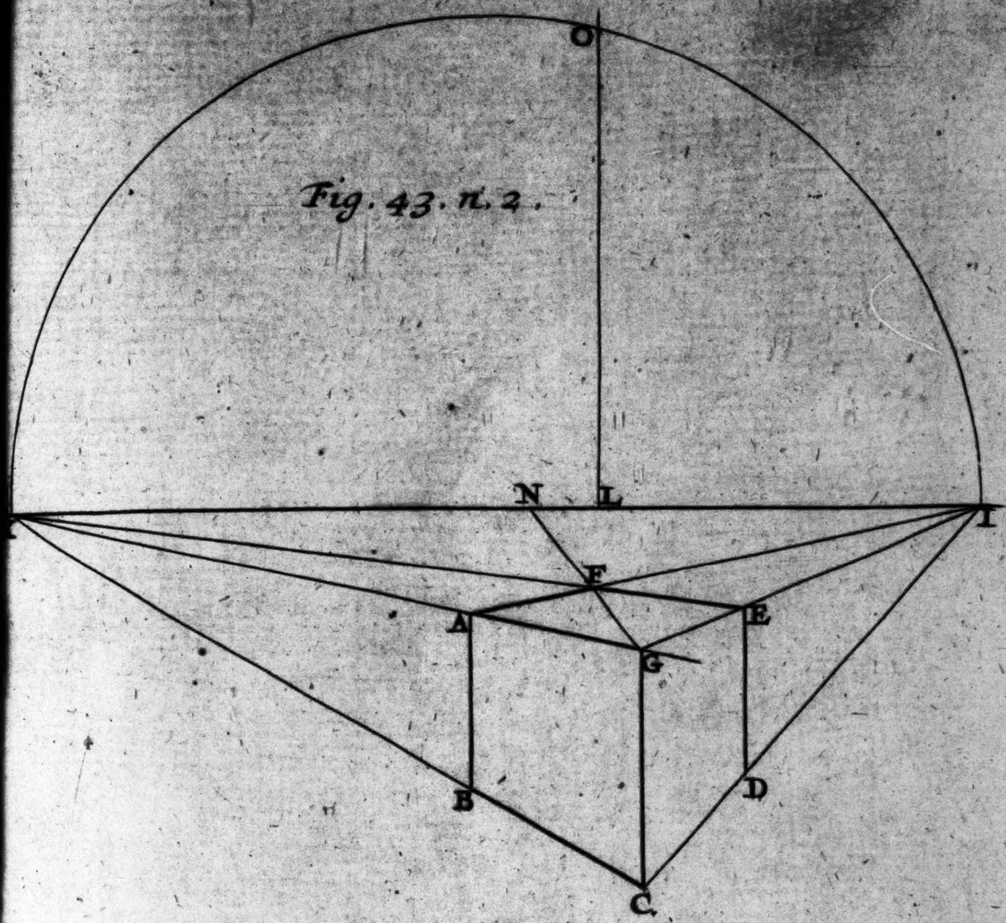

Fig. 43. n. 2.

ERRATA.

PAG. 2.	Lin. 11.	for	E T	read	E F	
3.	15.		A P		A B	
8.	15.		*c d f*		*e d f*	
	16.		*e d e f*		*c d e f*	
23.	1.		D T		D F	
28.	15. 16. 17. 18. 19. 20.	*every where*	O		R	*and on the contrary.*
			T		X	
31.	20.		R G		R B	
42.	23.		*in Cir.*		*in the Cir.*	

Richd: Beauvoir

+ Plus
— Minus
= Equal for Nums :: for Geometrical Proportion
× Multiplication

: Arethmetical Proportion disjunct Viz
7 . 3 : 13 . 9 is 7 as as much exceeds by 3
as 9 is by 13

:: 8 : 4 :: 30 : 15 . 8 is to 4 as, 30 to 15

∺ Geometrical proportion continued
Viz 2 . 4 . 8 . 16 &c

: 7 as much exceeds 3 as 13 exceeds
9